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On intuitionistic Fuzzy Asly Ideal of Ring

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Abstract

In this paper we tend to describe the notions of intuitionistic fuzzy asly ideal of ring indicated by (I. F.ASLY) ideal and, we will explore some properties and connections about this concept.

Key words: fuzzy set, intuitionistic fuzzy sub ring, intuitionistic fuzzy ideal.

1. Introduction

In 1965, Zadeh introduceed the notion of a fuzzy set [1]. In 1986 Rosenfeld applied this concept to group theory[2]. In 1986 Atanassov introduced the concept of intuitionistic fuzzy set

. Let A in a non-empty set X is an object having the form A= $\{(x, \mu_A(x), \lambda_A(x)) | x \in X\}$, where the functions $\mu_A : X \to [0,1]$ denote the degree of membership and $\lambda_A : X \to [0,1]$ the degree of non-membership of each element $x \in X$ to the set A and $0 \le \mu_A(x) + \lambda_A(x) \le 1$ for all $x \in X$[3]. In 1989 Biswas introduced the intuitionistic fuzzy subgroup and studied some of its properties [4]. In 2003 Banerjee and Basnet investigated intuitionistic fuzzy subrings and intuitionistic fuzzy ideal using intuitionistic fuzzy sets [5-7]. In this paper, we will recall some basic definitions. Let R be a ring, an (I.F.S) A= $\{(\lambda, \overline{\partial_A}(\lambda), \overline{\nabla_A}(\lambda)), \lambda \in R\}$ of R is said to be intuitionistic fuzzy subring means by (IFS) of R if

$$\overline{\partial_A}(\lambda - \omega) \ge \min\{\overline{\overline{\partial_A}}(\lambda), \overline{\partial_A}(\omega)\}, \overline{\partial}_A(\lambda\omega) \ge \min\{\overline{\partial_A}(\lambda), \overline{\partial_A}(\omega)\}, \overline{\nabla_A}(\lambda - \omega) \le \max\{\overline{\nabla_A}(\lambda), \overline{\nabla_A}(\omega)\}$$

and $\overline{\nabla_A}(\lambda\omega) \le \max\{\overline{\nabla_A}(\lambda), \overline{\nabla_A}(\omega)\}, \forall \lambda, \omega \in R$.[6,7].

In 2012, sharma P.K introduced the notion of intuitionistic fuzzy ideal by (I.F.I). Let:

A= $\{(\lambda, \overline{\partial_A}(\lambda), \overline{\nabla_A}(\lambda)), \lambda \in \mathrm{R}\}$ of a ring R if satisfies the four conditions,

$\overline{\partial_A}(\lambda - \omega) \ge \min\{\overline{\partial_A}(\lambda), \overline{\partial_A}(\omega)\}$, $\overline{\partial_A}(\lambda\omega) \ge \max\{\overline{\partial_A}(\lambda), \overline{\partial_A}(\omega)\}$,

$\overline{\nabla_A}(\lambda - \omega) \le \max\{\overline{\nabla_A}(\lambda), \overline{\nabla_A}(\omega)\}$ and $\overline{\nabla_A}(\lambda\omega) \le \min\{\overline{\nabla_A}(\lambda), \overline{\nabla_A}(\omega)\}$ [8].

2. Intuitionistic Fuzzy Asly Ideal

Definition (1)

Let A= $\{(ƛ, \overline{\partial_A}(ƛ), \overline{\nabla_A}(ƛ)), ƛ \in R\}$ be (IFS) of a ring R said to be an intuitionistic fuzzy asly ideal of a ring R means by (I. F.ASLY) ideal if and only if

1. $\overline{\partial_A}(ƛ+\omega) \le \overline{\partial_A}(ƛ)$
2. $\overline{\partial_A}(ƛ\omega) \ge \max\{\overline{\partial_A}(ƛ), \overline{\partial_A}(\omega)\}$
3. $\overline{\nabla_A}(ƛ+\omega) \ge \overline{\nabla_A}(\omega)\}$
4. $\overline{\nabla_A}(ƛ\omega) \le \min\{\overline{\nabla_A}(ƛ), \overline{\nabla_A}(\omega)\}. \forall ƛ, \omega \in R$

where $\overline{\partial_A}(ƛ): R \to [0,1], \overline{\nabla_A}(\omega): R \to [0,1].$

Example (2)

Let R be the set of 2×2 matrices over non negative integer Z

$$\overline{\partial_A}(ƛ) = \begin{cases} 0.8 & \text{if } ƛ = \begin{pmatrix} s & q \\ 0 & 0 \end{pmatrix} \text{ where } s, q \in Z^+/\{1,3\} \\ 0.1 & \text{otherwise} \end{cases}$$

$$\overline{\nabla_A}(ƛ) = \begin{cases} 0.2 & \text{if } ƛ = \begin{pmatrix} s & q \\ 0 & 0 \end{pmatrix} \text{ where } s, q \in Z^+/\{1,3\} \\ 0.9 & \text{otherwise} \end{cases}$$

Clearly $A = \{(ƛ, \overline{\partial_A}(ƛ), \overline{\nabla_A}(ƛ)), ƛ \in Z\}$ is an (I. F.ASLY) ideal .

Definition (3)

Let A= $\{(ƛ, \overline{\partial_A}(ƛ), \overline{\nabla_A}(ƛ)), ƛ \in R\}$, B $= \{(ƛ, \overline{\partial_B}(ƛ), \overline{\nabla_B}(ƛ)), ƛ \in R\}$ are any two (I. F.ASLY) ideals then their product is defined by:

$$\overline{\partial_A}(ƛ) . \overline{\partial_B}(ƛ) = \vee_{ƛ=\omega.s} [\overline{\partial_A}(\omega) \wedge \overline{\partial_B}(s)]$$

$$\overline{\nabla_A}(ƛ) . \overline{\nabla_B}(ƛ) = \wedge_{ƛ=\omega.s} [\overline{\nabla_A}(\omega) \vee \overline{\nabla_B}(s)], \forall ƛ, s, \omega \in R.$$

Definition (4)

Let A= $\{(ƛ, \overline{\partial_A}(ƛ), \overline{\nabla_A}(ƛ)), ƛ \in R\}$, B $= \{(ƛ, \overline{\partial_B}(ƛ), \overline{\nabla_B}(ƛ)), ƛ \in R\}$ are any (I. F.ASLY) ideals then their sum is defined by:

$A+B = \{ ƛ, \overline{\partial_A}(ƛ) + \overline{\partial_B}(ƛ), \overline{\nabla_A}(ƛ) + \overline{\nabla_B}(ƛ), ƛ \in R \}$. where

$$\overline{\partial_A}(ƛ) . \overline{\partial_B}(ƛ) = \vee_{ƛ=\omega+s} [\overline{\partial_A}(\omega) \wedge \overline{\partial_B}(s)]$$

$$\overline{\nabla_A}(ƛ) + \overline{\nabla_B}(ƛ) = \wedge_{ƛ=\omega+s} [\overline{\nabla_A}(\omega) \vee \overline{\nabla_B}(s)], \forall ƛ, \omega, s \in R.$$

Theorem (5)

Let A= $\{(ƛ, \overline{\partial_A}(ƛ), \overline{\nabla_A}(ƛ)), ƛ \in R\}$ be (I. F.ASLY) ideal of a ring R and let $A^* = \{(ƛ, \overline{\partial_A^*}(ƛ), \overline{\nabla_A^*}(ƛ)), ƛ \in R\}$ be the (IFS) of R is characterized by $\overline{\partial_A}^*(ƛ) = \overline{\partial_A}(ƛ) + 1 - \overline{\partial_A}(0)$, $\overline{\nabla_A^*}(ƛ) = \overline{\nabla_A}(ƛ) - \overline{\nabla_A}(0) + 1$, $\forall ƛ \in$ R then A^* is (I. F.ASLY) an ideal of R.

Proof

For all $ƛ \in R$ $\overline{\partial_A}^*(ƛ) = \overline{\partial_A}(ƛ) + 1 - \overline{\partial_A}(0)$, $\overline{\nabla_A^*}(ƛ) = \overline{\nabla_A}(ƛ) - \overline{\nabla_A}(0) + 1$

we have

$\overline{\partial^*_A}(\lambda+\omega)=\overline{\partial_A}(\lambda+\omega)+1-\overline{\partial_A}(0)$

$\le\overline{\partial_A}(\lambda)+1-\overline{\partial_A}(0)=\overline{\partial^*_A}(\lambda)$.......................(1)

$\overline{\partial^*_A}(\lambda\omega)=\overline{\partial_A}(\lambda\omega)+1-\overline{\partial_A}(0)$

$\ge\max\{\overline{\partial_A}(\lambda),\overline{\partial_A}(\omega)\}\ +1-\overline{\partial_A}(0)$

$\ge\max\{\overline{\partial_A}(\lambda)+1-\overline{\partial_A}(0),\overline{\partial_A}(\omega)+1-\overline{\partial_A}(0)\}$

$=\max\{\overline{\partial^*_A}(\lambda),\overline{\partial^*_A}(\omega)\}$..........................(2)

$\overline{\nabla^*_A}(\lambda+\omega)=\overline{\nabla_A}(\lambda+\omega)-\overline{\nabla_A}(0)+1$

$=\overline{\nabla_A}(\omega)\ -\overline{\nabla_A}(0)+1)=\overline{\nabla^*_A}(\omega)\}$....................................(3)

$\overline{\nabla^*_A}(\lambda\omega)=\overline{\nabla_A}(\lambda\omega)-\overline{\nabla_A}(0)+1$

$\le\min\{\overline{\nabla_A}(\lambda),\overline{\nabla_A}(\omega)\}\ -\overline{\nabla_A}(0)+1$

$=\min\{\overline{\nabla_A}(\lambda)-\overline{\nabla_A}(0)+1\ ,\overline{\nabla_A}(\omega)-\overline{\nabla_A}(0)+1\ \}$

$=\min\{\overline{\nabla^*_A}(\lambda),\overline{\nabla^*_A}(\omega)\}$................................ (4)

In forms (1),(2),(3) and (4), we have A^* is (I. F.ASLY) ideal of R

Theorem (6)

Let A be (I. F.ASLY) ideal of R and let

$\zeta:[0,\overline{\partial}_A(0)]\rightarrow[0,1]\ \ ,\upsilon:[0,\overline{\nabla}_A(0)]\rightarrow[0,1]$ be increasing functions, then (IFS)

$A_\zeta=\{(\lambda,\overline{\partial_{A\zeta}}(\lambda),\overline{\nabla_{A\zeta}}\ (\lambda)),\lambda\in R\ \}$

Means by $\overline{\partial_{A\zeta}}(\lambda)\ =\ \zeta(\ \overline{\partial_A}(\lambda))\ \ ,\overline{\nabla_{A\zeta}}\ (\lambda)=\zeta(\overline{\nabla_A}(\lambda))$ is (I. F.ASLY) ideal of R.

Proof

$\forall\lambda,\omega\in R$

$\overline{\partial_{A\zeta}}(\lambda+\omega)=\zeta\ (\overline{\partial_A}(\lambda+\omega))$

$\le\zeta\overline{\partial_A}(\lambda)=\overline{\partial_{A\zeta}}(\lambda)$.........................(1)

$\overline{\partial_{A\zeta}}(\lambda\omega)=\zeta(\overline{\partial_A}(\lambda\omega))$

$\ge\max\ \{\zeta(\overline{\partial_A}(\lambda)),\zeta(\overline{\partial_A}(\omega))\}$

$=\max\{\overline{\partial_{A\zeta}}(\lambda),\overline{\partial_{A\zeta}}(\omega)\ \}$............................(2)

$\overline{\nabla_{A\zeta}}(\lambda+\omega)=\zeta(\overline{\nabla_A}(\lambda+\omega))$

$\ge\zeta\overline{\nabla_A}(\omega)=\overline{\nabla_{A\zeta}}(\omega)\}$ ....(3)

$\overline{\nabla_{A\zeta}}(\lambda\omega)=\zeta(\ \overline{\nabla_A}(\lambda\omega))$

$\le\min\ \{\zeta(\overline{\nabla_A}(\lambda)),\zeta(\overline{\nabla_A}(\omega))\}$

$=\min\ \{\overline{\nabla_{A\zeta}}(\lambda),\overline{\nabla_{A\zeta}}(\omega)\}$ ....(4)

In forms (1),(2),(3) and (4), we get A_f an (I. F.ASLY) ideal of R.

Theorem (7)

If $\{B_j\big|j\in\eta\}$ is a family of (I. F.ASLY) ideals of R, then $\prod_{j\in J}B_j$ is (I. F.ASLY) ideal of R.

Proof

Let $\{B_j \mid j \in \eta\}$ be a family of (I. F.ASLY) ideals of R, for all $\lambda, \omega \in$ R . We have

$$\prod_{j\in\eta} B_j = \{(\lambda, \prod_{j\in\eta} \overline{\partial_{B_j}}(\lambda), \prod_{j\in\eta} \overline{\nabla_{B_j}}(\lambda)), \lambda \in R \}$$

$$\prod_{j\in\eta} \overline{\partial}_{B_j}(\lambda+\omega) \le \inf_{j\in\eta} \{\overline{\partial}_{B_j}(\lambda+\omega)\}$$

$$\le \inf_{j\in\eta} \overline{\partial}_{B_j}(\lambda) = \prod_{j\in\eta} \overline{\partial}_{B_j}(\lambda) \text{..........................(1)}$$

$$\prod_{j\in\eta} \overline{\partial}_{B_j}(\lambda\omega) \ge \sup_{j\in\eta} (\max\{\overline{\partial}_{B_j}(\lambda), \overline{\partial}_{B_j}(\omega)\})$$

$$= \max\{\sup_{j\in\eta} \overline{\partial}_{B_j}(\lambda), \sup_{j\in\eta} \overline{\partial}_{B_j}(\omega)\}$$

$$= \max\{\prod_{j\in\eta} \overline{\partial}_{B_j}(\lambda), \prod_{j\in\eta} \overline{\partial}_{B_j}(\omega)\} \text{.............................(2)}$$

$$\prod_{j\in\eta} \overline{\nabla}_{B_j}(\lambda+\omega) \ge \inf_{j\in\eta} \overline{\nabla}_{B_j}(\omega) = \prod_{j\in\eta} \overline{\nabla}_{B_j}(\omega) \text{...........................(3)}$$

$$\prod_{j\in\eta} \overline{\nabla}_{B_j}(\lambda\omega) \le \inf_{j\in\eta} (\min\{\overline{\nabla}_{B_j}(\lambda), \overline{\nabla}_{B_j}(\omega)\})$$

$$= \min\{\inf_{j\in\eta} \overline{\nabla}_{B_j}(\lambda), \inf_{j\in\eta} \overline{\nabla}_{B_j}(\omega)\}$$

$$= \min\{\prod_{j\in\eta} \overline{\nabla}_{B_j}(\lambda), \prod_{j\in\eta} \overline{\nabla}_{B_j}(\omega)\} \text{.............................(4)}$$

In forms (1),(2),(3) and (4) $\prod_{j\in\eta} B_j = \{ \lambda, \prod_{j\in\eta} \overline{\partial_{B_j}}(\lambda), \prod_{j\in\eta} \overline{\nabla}_{B_j}(\lambda), \lambda \in R \}$ is (I. F.ASLY) ideal of R.

Theorem (8)

Let A= $\{(\lambda, \overline{\partial_A}(\lambda), \overline{\nabla_A}(\lambda)), \lambda \in R \}$ be a (I. F.ASLY) ideal of R, then

1. One of $\overline{\partial_A}(\lambda), \overline{\partial_A}(\omega), \overline{\partial_A}(\lambda.\omega)$ at least two are equal.
2. One of $\overline{\nabla_A}(\lambda), \overline{\nabla_A}(\omega), \overline{\nabla_A}(\lambda.\omega)$ at least two are equal.

Proof 1

If $\overline{\partial_A}(\lambda) \ne \overline{\partial_A}(\omega)$, so we have two cases:

Case 1: if $\overline{\partial_A}(\lambda) \prec \overline{\partial_A}(\omega)$

$\overline{\partial_A}(\omega.\lambda) = \max\{\overline{\partial_A}(\lambda), \overline{\partial_A}(\omega)\}$

Then $\overline{\partial_A}(\lambda.\omega) = \overline{\partial_A}(\omega)$.

Case 2: if $\overline{\partial_A}(\lambda) \succ \overline{\partial_A}(\omega)$

$\overline{\partial_A}(\lambda.\omega) = \max\{\overline{\partial_A}(\lambda), \overline{\partial_A}(\omega)\}$

Then $\overline{\partial_A}(\lambda.\omega) = \overline{\partial_A}(\lambda)$.

Proof 2

If $\overline{\nabla_A}(\lambda) \ne \overline{\nabla_A}(\omega)$

Either $\overline{\nabla_A}(\lambda) \prec \overline{\nabla_A}(\omega)$

$\overline{\nabla_A}(\lambda.\omega) = \min\{\overline{\nabla_A}(\lambda), \overline{\nabla_A}(\omega)\}$

$\overline{\nabla_A}(\lambda.\omega) = \overline{\nabla_A}(\lambda)$.

Or $\overline{\nabla_A}(\hat{\lambda}) \succ \overline{\nabla_A}(\omega)$

$\overline{\nabla_A}(\hat{\lambda}.\omega) = \min\{\overline{\nabla_A}(\hat{\lambda}), \overline{\nabla_A}(\omega)\}$

$\overline{\nabla_A}(\hat{\lambda}.\omega) = \overline{\nabla_A}(\omega).$

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