IHJPAS. 36 (3) 2023

Ibn Al-Haitham Journal for Pure and Applied Sciences
Journal homepage: jih.uobaghdad.edu.iq

Score for the group $\mathcal{SL}(2,3^8)$

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Article history: Received 2022, Accepted November 20٢2, Published in July 2023.

doi.org/10.30526/36.3.3017

Abstract

The set of all (n×n) non-singular matrices over the field F. And this set forms a group under the operation of matrix multiplication. This group is called the general linear group of dimension n over the field F, denoted by $\mathrm{GL}(n,F)$. The determinant of these matrices is a homomorphism from $\mathrm{GL}(n,F)$ into F* and the kernel of this homomorphism was the special linear group and denoted by $\mathcal{SL}(n,F)$. Thus $\mathcal{SL}(n,F)$ is the subgroup of $\mathrm{GL}(n,F)$ which contains all matrices of determinant one. The rational valued characters of the rational representations are written as a linear combination of the induced characters for the groups $\mathcal{SL}(2,3^8)$ discussed in this paper. We find the Artin indicator for this group after studying the rational valued characters of the rational representations and the induced characters.

Keywords: Rational character table, Induced characters table, Artin indicator.

1. Introduction

The group of all matrices of determinant 1 is (*n*,F), [1 and 2], and researchers in [3] defined the representation of the group. Authors in [4] survey and got the calculation for the groups SL(2, U), U = 31 and 37. We apply the same idea in [4] to compute the character table of rational representations for the group. Also, we apply the same idea in [5] to compute the Artin indicator for this group.

In this work, we count all cyclic subgroups, the Artin indicator, the rationally valued characters of the rational representations, and the induced characters for the group.

2. Basic Definitions and Facts

Theorem 2.1: [1]

$| \mathcal{SL}(2, q^n) | = q^n \ (q^{2n} - 1)$.

Definition 2.4: [1]

Let H be a cyclic subgroup of group G, and ϕ be a class function of H. Then

$$\phi^{\uparrow G}(g) = \frac{|C_G(g)|}{|C_H(g)|} \sum_{i=1}^{m} \phi(x_i)$$

Definition 2.5: [6]

The character induced from the principal character of cyclic subgroups of G is called Artin character.

Definition 2.6: [6]

Let G be a finite group and let χ be any rational valued character on G. The smallest positive number n such that,

$$n\chi = \sum_c a_c \phi_c$$

Where $a_c \in Z$ and ϕ_c is Artin character called the Artin exponent of G and is denoted by A (G).

3. The Results

Authors in [7-10] studied the character table of rational representations for the group $\mathcal{SL}(2, p)$. we apply that idea and compute the character table of rational representations for the group $\mathcal{SL}(2, 3^8)$. Also we use the same idea in [5] to compute the Artin indicator for this group.

The character table of rational representations for the group $\mathcal{SL}(2, 3^8)$ is:

C_n	1	z	c = d	zc = zd	a^1	a^2	a^4	a^5	a^8	a^{10}	a^{16}	a^{20}	a^{32}	a^{40}	a^{41}
$\|C_n\|$	1	1	21523360	21523360	43053282	43053282	43053282	43053282	43053282	43053282	43053282	43053282	43053282	43053282	43053282
$\|C_G(g)\|$	282429529920	282429529920	13122	13122	6560	6560	6560	6560	6560	6560	6560	6560	6560	6560	6560
1_G	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
ψ	6561	6561	0	0	1	1	1	1	1	1	1	1	1	1	1
χ_1	16798720	-16798720	2560	-2560	0	0	0	0	0	0	-40960	0	40960	0	0
χ_2	4199680	4199680	640	640	0	0	0	0	-5120	0	5120	0	-25600	20480	0
χ_4	2099840	2099840	320	320	0	0	-1280	0	1280	0	-6400	5120	6400	-5120	0
χ_5	4199680	-4199680	640	-640	0	0	0	0	0	0	10240	-20480	-10240	20480	0
χ_8	1049920	1049920	160	160	0	-1280	320	0	-3200	1280	1600	-1280	0	0	0
χ_{10}	1049920	1049920	160	160	0	0	0	0	1280	-2560	-1280	2560	0	0	0
χ_{16}	524960	524960	80	80	-80	80	-400	320	400	-320	0	0	0	3200	3200
χ_{20}	524960	524960	80	80	0	0	320	-640	-320	640	0	0	3200	0	0
χ_{32}	524960	524960	80	80	80	-400	400	-320	0	0	0	3200	6400	-3200	-3200
χ_{40}	262480	262480	40	40	0	80	-80	160	0	0	800	0	-800	0	0
χ_{41}	419968	-419968	64	-64	0	0	0	0	0	0	1024	0	-1024	0	0
χ_{80}	131240	131240	20	20	20	-20	0	0	200	0	-200	0	-400	-800	-800
χ_{82}	104992	104992	16	16	0	0	0	0	128	0	-128	0	-256	-512	-512
χ_{160}	131240	131240	20	20	-20	0	200	0	-200	0	-400	-800	800	800	800
χ_{164}	52496	52496	8	8	0	0	32	0	-32	0	-64	-128	128	128	128
χ_{205}	104992	-104992	16	-16	0	0	0	128	0	-128	-256	-256	256	256	256
χ_{328}	26248	26248	4	4	0	8	-8	0	-16	-32	32	32	32	32	32
χ_{410}	26248	26248	4	4	0	0	0	-16	-32	-32	32	32	32	32	32
χ_{656}	13124	13124	2	2	2	-2	-4	8	8	8	8	8	8	8	8
χ_{820}	13124	13124	2	2	0	0	-8	-8	8	8	8	8	8	8	8
χ_{1312}	13124	13124	2	2	-2	-4	8	8	8	8	8	8	8	8	8
χ_{1640}	6562	6562	1	1	0	-2	2	2	2	2	2	2	2	2	2
θ_1	20152320	-20152320	-3072	3072	0	0	0	0	0	0	0	0	0	0	0
θ_2	10076160	10076160	-1536	-1536	0	0	0	0	0	0	0	0	0	0	0
θ_{17}	1259520	-1259520	-192	192	0	0	0	0	0	0	0	0	0	0	0
θ_{34}	629760	629760	-96	-96	0	0	0	0	0	0	0	0	0	0	0
θ_{193}	104960	-104960	-16	16	0	0	0	0	0	0	0	0	0	0	0
θ_{386}	52480	52480	-8	-8	0	0	0	0	0	0	0	0	0	0	0
ξ	6562	6562	1	1	-2	2	2	-2	2	2	2	2	2	2	-2
η	13120	-13120	-1	1	0	0	0	0	0	0	0	0	0	0	0

C_{it}	a^{80}	a^{82}	a^{160}	a^{164}	a^{205}	a^{328}	a^{410}	a^{656}	a^{820}	a^{1312}	a^{1640}	b^{1}	b^{2}	b^{17}	b^{34}	b^{193}	b^{386}
$\|C_{it}\|$	43053282	43053282	43053282	43053282	43053282	43053282	43053282	43053282	43053282	43053282	43053282	43040160	43040160	43040160	43040160	43040160	43040160
$\|C_G(g)\|$	6560	6560	6560	6560	6560	6560	6560	6560	6560	6560	6560	6562	6562	6562	6562	6562	6562
1_G	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Ψ	1	1	1	1	1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1
χ_1	163840	0	-163840	0	0	0	0	1638400	0	-1638400	0	0	0	0	0	0	0
χ_2	-20480	0	0	0	0	204800	0	-204800	0	-409600	-819200	0	0	0	0	0	0
χ_4	0	0	51200	51200	0	-51200	0	-102400	-204800	204800	204800	0	0	0	0	0	0
χ_5	0	0	0	0	0	0	0	102400	0	409600	409600	0	0	0	0	0	0
χ_8	12800	12800	-12800	-12800	0	-25600	-51200	51200	51200	51200	51200	0	0	0	0	0	0
χ_{10}	0	0	0	0	-25600	-51200	-51200	51200	51200	51200	51200	0	0	0	0	0	0
χ_{16}	-3200	-3200	-6400	-6400	-12800	12800	12800	12800	12800	12800	12800	0	0	0	0	0	0
χ_{20}	0	0	-12800	-12800	-12800	12800	12800	12800	12800	12800	12800	0	0	0	0	0	0
χ_{32}	-6400	-6400	12800	12800	12800	12800	12800	12800	12800	12800	12800	0	0	0	0	0	0
χ_{40}	-3200	-3200	3200	3200	3200	3200	3200	3200	3200	3200	3200	0	0	0	0	0	0
χ_{41}	-4096	-4096	4096	4096	4096	4096	4096	4096	4096	4096	4096	0	0	0	0	0	0
χ_{80}	800	800	800	800	800	800	800	800	800	800	800	0	0	0	0	0	0
χ_{82}	512	512	512	512	512	512	512	512	512	512	512	0	0	0	0	0	0
χ_{160}	800	800	800	800	800	800	800	800	800	800	800	0	0	0	0	0	0
χ_{164}	128	128	128	128	128	128	128	128	128	128	128	0	0	0	0	0	0
χ_{205}	256	256	256	256	256	256	256	256	256	256	256	0	0	0	0	0	0
χ_{328}	32	32	32	32	32	32	32	32	32	32	32	0	0	0	0	0	0
χ_{410}	32	32	32	32	32	32	32	32	32	32	32	0	0	0	0	0	0
χ_{656}	8	8	8	8	8	8	8	8	8	8	8	0	0	0	0	0	0
χ_{820}	8	8	8	8	8	8	8	8	8	8	8	0	0	0	0	0	0
χ_{1312}	8	8	8	8	8	8	8	8	8	8	8	0	0	0	0	0	0
χ_{1640}	2	2	2	2	2	2	2	2	2	2	2	0	0	0	0	0	0
θ_1	0	0	0	0	0	0	0	0	0	0	0	3072	-3072	-49152	49152	-589824	589824
θ_2	0	0	0	0	0	0	0	0	0	0	0	-1536	0	24576	49152	294912	0
θ_{17}	0	0	0	0	0	0	0	0	0	0	0	-192	192	-3072	0	36864	-36864
θ_{34}	0	0	0	0	0	0	0	0	0	0	0	96	192	0	-9216	-18432	-18432
θ_{193}	0	0	0	0	0	0	0	0	0	0	0	-16	16	256	-256	-256	-256
θ_{386}	0	0	0	0	0	0	0	0	0	0	0	-8	0	-128	-128	-128	-128
ξ	2	2	2	2	-2	2	2	2	2	2	2	0	0	0	0	0	0
η	0	0	0	0	0	0	0	0	0	0	0	4	-4	4	-4	4	-4

This group has 32 cyclic subgroups generated by the conjugacy classes of the group. The induced character table for this group is:

C_{it}	1	z	$c=d$	$zc=zd$	a^{1}	a^{2}	a^{4}	a^{5}	a^{8}	a^{10}	a^{16}	a^{20}	a^{32}	a^{40}	a^{41}
$\|C_{it}\|$	1	1	21523360	21523360	43053282	43053282	43053282	43053282	43053282	43053282	43053282	43053282	43053282	43053282	43053282
$\|C_G(g)\|$	282429529920	282429529920	13122	13122	6560	6560	6560	6560	6560	6560	6560	6560	6560	6560	6560
Φ_1	282429529920	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Φ_2	35303691240	35303691240	0	0	0	0	0	0	0	0	0	0	0	0	0
Φ_3	43046720	0	2	0	0	0	0	0	0	0	0	0	0	0	0
Φ_4	43046720	21523360	2	2	0	0	0	0	0	0	0	0	0	0	0
Φ_5	43053282	86106564	0	0	2	0	0	0	0	0	0	0	0	0	0
Φ_6	86106564	172213128	0	0	0	4	0	0	0	0	0	0	0	0	0
Φ_7	172213128	0	0	0	0	0	8	0	0	0	0	0	0	0	0
Φ_8	430532820	861065640	0	0	0	0	0	10	0	0	0	0	0	0	0
Φ_9	34426256	0	0	0	0	0	0	0	16	0	0	0	0	0	0
Φ_{10}	430532820	0	0	0	0	0	0	0	0	20	0	0	0	0	0
Φ_{11}	6888521512	0	0	0	0	0	0	0	0	0	32	0	0	0	0
Φ_{12}	861065640	0	0	0	0	0	0	0	0	0	0	40	0	0	0
Φ_{13}	1377705024	0	0	0	0	0	0	0	0	0	0	0	64	0	0
Φ_{14}	1722131280	0	0	0	0	0	0	0	0	0	0	0	0	80	0
Φ_{15}	1765184562	3530369124	0	0	0	0	0	0	0	0	0	0	0	0	82
Φ_{16}	3444262560	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Φ_{17}	3530369124	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Φ_{18}	6888525120	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Φ_{19}	7060738248	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Φ_{20}	8825922810	17651845620	0	0	0	0	0	0	0	0	0	0	0	0	0
Φ_{21}	14121476490	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Φ_{22}	17651845620	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Φ_{23}	28242952992	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Φ_{24}	35303691240	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Φ_{25}	56485905984	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Φ_{26}	70607382480	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Φ_{27}	860803020	1721606040	0	0	0	0	0	0	0	0	0	0	0	0	0
Φ_{28}	1721606040	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Φ_{29}	42129347980	84358695960	0	0	0	0	0	0	0	0	0	0	0	0	0
Φ_{30}	29267302680	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Φ_{31}	16613458286	332266996572	0	0	0	0	0	0	0	0	0	0	0	0	0
Φ_{32}	33226996572	0	0	0	0	0	0	0	0	0	0	0	0	0	0

C_{tt}	a^{80}	a^{82}	a^{160}	a^{164}	a^{205}	a^{328}	a^{410}	a^{656}	a^{820}	a^{1312}	a^{1640}	b^{1}	b^{2}	b^{7}	b^{34}	b^{193}	b^{386}
$\lvert C_{tt}\rvert$	43053282	43053282	43053282	43053282	43053282	43053282	43053282	43053282	43053282	43053282	43053282	43040160	43040160	43040160	43040160	43040160	43040160
$\lvert C_{tt}(g)\rvert$	6560	6560	6560	6560	6560	6560	6560	6560	6560	6560	6560	6562	6562	6562	6562	6562	6562
Φ_1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Φ_2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Φ_3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Φ_4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Φ_5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Φ_6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Φ_7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Φ_8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Φ_9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Φ_{10}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Φ_{11}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Φ_{12}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Φ_{13}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Φ_{14}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Φ_{15}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Φ_{16}	160	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Φ_{17}	0	164	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Φ_{18}	0	0	320	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Φ_{19}	0	0	0	328	0	0	0	0	0	0	0	0	0	0	0	0	0
Φ_{20}	0	0	0	0	410	0	0	0	0	0	0	0	0	0	0	0	0
Φ_{21}	0	0	0	0	0	656	0	0	0	0	0	0	0	0	0	0	0
Φ_{22}	0	0	0	0	0	0	820	0	0	0	0	0	0	0	0	0	0
Φ_{23}	0	0	0	0	0	0	0	1312	0	0	0	0	0	0	0	0	0
Φ_{24}	0	0	0	0	0	0	0	0	1640	0	0	0	0	0	0	0	0
Φ_{25}	0	0	0	0	0	0	0	0	0	2624	0	0	0	0	0	0	0
Φ_{26}	0	0	0	0	0	0	0	0	0	0	3280	0	0	0	0	0	0
Φ_{27}	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0
Φ_{28}	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0
Φ_{29}	0	0	0	0	0	0	0	0	0	0	0	0	0	34	0	0	0
Φ_{30}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	68	0	0
Φ_{31}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	386	0
Φ_{32}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	772

Hence, the rational valued characters in the first table are written as a linear combination of induced characters in the second table.

$$1 = \frac{1}{772}\Phi_{32} + \frac{1}{386}\Phi_{31} + \frac{1}{68}\Phi_{30} + \frac{1}{14}\Phi_{29} + \frac{1}{2}\Phi_{28} + \frac{1}{2}\Phi_{27} + \frac{1}{3280}\Phi_{26} + \frac{1}{2624}\Phi_{25} + \frac{1}{1640}\Phi_{24} + \frac{1}{1312}\Phi_{23} + \frac{1}{820}\Phi_{22} + \frac{1}{656}\Phi_{21} + \frac{1}{410}\Phi_{20} + \frac{1}{328}\Phi_{19} + \frac{1}{320}\Phi_{18} + \frac{1}{164}\Phi_{17} + \frac{1}{160}\Phi_{16} + \frac{1}{82}\Phi_{15} + \frac{1}{80}\Phi_{14} + \frac{1}{64}\Phi_{13} + \frac{1}{40}\Phi_{12} + \frac{1}{32}\Phi_{11} + \frac{1}{20}\Phi_{10} + \frac{1}{16}\Phi_9 + \frac{1}{10}\Phi_8 + \frac{1}{8}\Phi_7 + \frac{1}{4}\Phi_6 + \frac{1}{2}\Phi_5 + \frac{1}{2}\Phi_4 - \frac{0.10472284}{35303691240}\Phi_2 - \frac{7.6371094131}{282429529920}\Phi_1,$$

$$\Psi = -\frac{1}{772}\Phi_{32} - \frac{1}{386}\Phi_{31} - \frac{1}{68}\Phi_{30} - \frac{1}{14}\Phi_{29} - \frac{1}{2}\Phi_{28} - \frac{1}{2}\Phi_{27} + \frac{1}{3280}\Phi_{26} + \frac{1}{2624}\Phi_{25} + \frac{1}{1640}\Phi_{24} + \frac{1}{1312}\Phi_{23} + \frac{1}{820}\Phi_{22} + \frac{1}{656}\Phi_{21} + \frac{1}{410}\Phi_{20} + \frac{1}{328}\Phi_{19} + \frac{1}{320}\Phi_{18} + \frac{1}{164}\Phi_{17} + \frac{1}{160}\Phi_{16} + \frac{1}{82}\Phi_{15} + \frac{1}{80}\Phi_{14} + \frac{1}{64}\Phi_{13} + \frac{1}{40}\Phi_{12} + \frac{1}{32}\Phi_{11} + \frac{1}{20}\Phi_{10} + \frac{1}{16}\Phi_9 + \frac{1}{10}\Phi_8 + \frac{1}{8}\Phi_7 + \frac{1}{4}\Phi_6 + \frac{1}{2}\Phi_5 + \frac{0.0199021653}{35303691240}\Phi_2 + \frac{0.0222530817}{282429529920}\Phi_1,$$

$$\chi_1 = -624.39024\Phi_{25} + 1248.78049 - 512\Phi_{18} + 1024\Phi_{16} + 640\Phi_{13} - 1280\Phi_{11} - 1280\Phi_4 + 2560\Phi_3 + \frac{27533102080}{35303691240}\Phi_2 + \frac{33597440}{282429529920}\Phi_1,$$

$$\chi_2 = -249.75609\Phi_{26} - 156.09756\Phi_{25} - 156.09756\Phi_{23} + 312.195122\Phi_{21} - 128\Phi_{16} + 256\Phi_{14} - 400\Phi_{13} + 160\Phi_{11} - 320\Phi_9 + 320\Phi_4 - \frac{6883275520}{35303691240}\Phi_2 + \frac{26996130377988}{282429529920}\Phi_1,$$

$\chi_4 =$ $62.43902\Phi_{26} + 78.04878\Phi_{25} - 124.87805\ \Phi_{24} - 78.04878\Phi_{23} - 78.04878\Phi_{21} + 165.09786\Phi_{19} + 160\Phi_{18} - 64\Phi_{14} + 100\Phi_{13} + 128\Phi_{12} - 200\ \Phi_{11} + 80\ \Phi_9 - 160\Phi_7 + 160\Phi_4 - \frac{342273760}{35303691240}\Phi_2 - \frac{12.0817024875}{282429529920}\Phi_1,$

$\chi_5 = 124.87805\Phi_{26} + 156.0956\Phi_{25} + 78.04871\ \Phi_{23} + 256\Phi_{14} - 160\Phi_{13} - 512\Phi_{12} + 320\Phi_{11} - 320\Phi_4 - 640\ \Phi_3 + \frac{6883275520}{35303691240}\Phi_2 - \frac{19859604412055}{282429529920}\Phi_1,$

$\chi_8 = 15.60976\Phi_{26} + 19.51219\Phi_{25} + 31.21951\ \Phi_{24} + 39.02439\Phi_{23} - 62.43904\Phi_{22} - 39.02439\Phi_{21} - 39.02439\ \Phi_{19} - 40\Phi_{18} + 78.04878\Phi_{17} + 80\ \Phi_{16} - 32\Phi_{12} + 50\Phi_{11} + 64\ \Phi_{10} - 200\Phi_9 + 40\Phi_7 - 320\Phi_6 + 80\Phi_4 + \frac{53387382080}{35303691240}\Phi_2 - \frac{9.7621928819}{282429529920}\Phi_1,$

$\chi_{10} = 15.60976\Phi_{26} + 19.51219\Phi_{25} + 31.21951\ \Phi_{24} + 39.02439\Phi_{23} - 62.43904\Phi_{22} - 78.04878\Phi_{21} - 62.43902\ \Phi_{20} + 64\Phi_{12} - 40\Phi_{11} - 128\ \Phi_{10} + 80\Phi_9 + 80\Phi_4 + \frac{31.1707666868}{35303691240}\Phi_2 - \frac{29791149178961}{282429529920}\Phi_1,$

$\chi_{16} = 3.90244\Phi_{26} + 4.87805\Phi_{25} + 7.80488\ \Phi_{24} + 9.75609\Phi_{23} + 15.60976\Phi_{22} + 19.51219\Phi_{21} - 31.21951\ \Phi_{20} - 19.51219\Phi_{19} - 20\Phi_{18} - 19.51219\Phi_{17} - 20\Phi_{16} + 39.02439\Phi_{15} + 40\Phi_{14} - 16\Phi_{10} + 25\Phi_9 + 32\Phi_8 - 50\Phi_7 + 20\Phi_6 - 40\Phi_5 + 40\Phi_4 + \frac{10.9024565385}{35303691240}\Phi_2 - \frac{13972510063211}{282429529920}\Phi_1,$

$\chi_{20} = 3.90244\Phi_{26} + 4.87805\Phi_{25} + 7.80488\ \Phi_{24} + 9.75609\Phi_{23} + 15.60976\Phi_{22} + 19.51219\Phi_{21} - 31.21951\ \Phi_{20} - 39.02439\Phi_{19} - 40\Phi_{18} + 50\Phi_{13} - 32\Phi_{10} - 20\Phi_9 - 64\Phi_8 + 40\Phi_7 + 40\Phi_4 - \frac{17.1463589531}{35303691240}\Phi_2 - \frac{8.3446739192}{282429529920}\Phi_1,$

$\chi_{32} = 3.90244\Phi_{26} + 4.87805\Phi_{25} + 7.80488\ \Phi_{24} + 9.75609\Phi_{23} + 15.60976\Phi_{22} + 19.51219\Phi_{21} + 31.21951\ \Phi_{20} + 39.02439\Phi_{19} + 40\Phi_{18} - 39.02439\Phi_{17} - 40\Phi_{16} - 39.02439\Phi_{15} - 40\Phi_{14} + 100\Phi_{13} + 80\Phi_{12} - 32\Phi_8 + 50\Phi_7 - 100\Phi_6 + 40\Phi_4 + \frac{19.7805052702}{35303691240}\Phi_2 - \frac{5.0640237849}{282429529920}\Phi_1,$

$\chi_{40} = 0.97561\Phi_{26} + 1.21951\Phi_{25} + 1.95122\ \Phi_{24} + 2.43902\Phi_{23} + 3.90244\Phi_{22} + 4.87805\Phi_{21} + 7.80488\ \Phi_{20} + 9.75609\Phi_{19} + 10\Phi_{18} - 19.5119\Phi_{17} - 20\Phi_{16} - 12.5\Phi_{13} + 25\Phi_{11} + 16\Phi_8 - 10\Phi_7 + 20\Phi_6 + 20\Phi_4 - \frac{4.2926764675}{35303691240}\Phi_2 - \frac{0.7012190701}{282429529920}\Phi_1,$

$\chi_{41} = 1.24878\Phi_{26} + 1.56098\Phi_{25} + 2.49756\ \Phi_{24} + 3.12195\Phi_{23} + 4.99512\Phi_{22} + 6.24390\Phi_{21} + 9.99024\ \Phi_{20} + 12.4878\Phi_{19} + 12.8\Phi_{18} - 24.97561\Phi_{17} - 25.6\Phi_{16} - 16\Phi_{13} + 32\Phi_{11} - 32\Phi_4 + 64\Phi_3 + \frac{3.3756226747}{35303691240}\Phi_2 + \frac{87353388074774}{282429529920}\Phi_1,$

$\chi_{80} = 0.24390\Phi_{26} + 0.30488\Phi_{25} + 0.48780\Phi_{24} + 0.60976\Phi_{23} + 0.97561\Phi_{22} + 1.21951\Phi_{21} + 1.95129\Phi_{20} + 2.43902\Phi_{19} + 2.5\Phi_{18} + 4.87804\Phi_{17} + 5\Phi_{16} - 9.75609\Phi_{15}$

$- \ 10\Phi_{14} \ - 6.25\Phi_{13} \ - 6.25\Phi_{11} \ + \ 12.5\Phi_9 \ - 5\Phi_6 + 10\Phi_5 + \ 10\Phi_4 \ - \frac{0.0061289142}{35303691240}\Phi_2 - \frac{0.4641737119}{282429529920}\Phi_1,$

$\chi_{82} = 0.15609\Phi_{26} + 0.19512\Phi_{25} + 0.31219 \ \Phi_{24} + 0.39024\Phi_{23} + 0.62439\Phi_{22} + 0.78049\Phi_{21} + 1.24878 \ \Phi_{20} + 1.56098\Phi_{19} + 1.6\Phi_{18} + 3.12195\Phi_{17} + 3.2\Phi_{16} - 6.24390\Phi_{15} - 6.4\Phi_{14} - 4\Phi_{13} - 4\Phi_{11} + 8\Phi_9 + 8\Phi_4 - \frac{6888420128}{35303691240}\Phi_2 - \frac{1110540858122}{282429529920}\Phi_1,$

$\chi_{160} = 0.24390\Phi_{26} + 0.30488\Phi_{25} + 0.48780 \ \Phi_{24} + 0.60976\Phi_{23} + 0.97561\Phi_{22} + 1.21951\Phi_{21} + 1.95129 \ \Phi_{20} + 2.43902\Phi_{19} + 2.5\Phi_{18} + 4.87804\Phi_{17} + 5\Phi_{16} + 9.75609\Phi_{15} + 10\Phi_{14} + 12.5\Phi_{13} \ - 20\Phi_{12} \ - 12.5\Phi_{11} \ - 12.55\Phi_9 + 25\Phi_7 \ - 10\Phi_5 \ + 10\Phi_4 \ + \frac{0.0182613297}{35303691240}\Phi_2 - \frac{0.6193153408}{282429529920}\Phi_1,$

$\chi_{164} = 0.03902\Phi_{26} + 0.04878\Phi_{25} + 0.07805\Phi_{24} \ + 0.09756\Phi_{23} + 0.15609\Phi_{22} + 0.19512 \ + 0.31295\Phi_{20} \ + 0.39024\Phi_{19} + 0.4\Phi_{18} + 0.78049\Phi_{17} + 0.8\Phi_{16} + 1.56098\Phi_{15} \ + 1.6\Phi_{14} + 2\Phi_{13} - 3.2\Phi_{12} - 2\Phi_{11} - 2\Phi_9 + 4\Phi_7 \ + 4\Phi_4 - \frac{0.3150101657}{35303691240}\Phi_2 - \frac{0.112476513}{282429529920}\Phi_1,$

$\chi_{205} = 0.07804\Phi_{26} + 0.097561\Phi_{25} + 0.15609\Phi_{24} + 0.19512\Phi_{23} + 0.31219\Phi_{22} + 0.39024\Phi_{21} + 0.62439 \ \Phi_{20} + 0.78049\Phi_{19} + 0.8\Phi_{18} + 1.56098\Phi_{17} + 1.6\Phi_{16} + 3.12195\Phi_{15} + 3.2\Phi_{14} + 4\Phi_{13} \ - 6.4\Phi_{12} \ - 8\Phi_{11} - 6.4\Phi_{10} + \ 12.8\Phi_8 \ - \ 8\Phi_4 \ + \ 16\Phi_3 - \frac{0.9243937175}{35303691240}\Phi_2 - \frac{0.3051781266}{282429529920}\Phi_1,$

$\chi_{328} = 0.00976\Phi_{26} + 0.01219\Phi_{25} + 0.01951\Phi_{24} + 0.02439\Phi_{23} + 0.03902\Phi_{22} + 0.04878\Phi_{21} + 0.07805\Phi_{20} + 0.09756\Phi_{19} + 0.1\Phi_{18} + 0.19512\Phi_{17} + 0.2\Phi_{16} + 0.39024\Phi_{15} + 0.4\Phi_{14} + 0.5\Phi_{13} \ + \ 0.78049\Phi_{12} + \Phi_{11} \ - 1.6\Phi_{10} \ - \Phi_9 \ - \Phi_7 + \ 2\Phi_6 + \ 2\Phi_4 - \frac{0.0890236804}{35303691240}\Phi_2 - \frac{0.0467383892}{282429529920}\Phi_1,$

$\chi_{410} = 0.00976\Phi_{26} + 0.01219\Phi_{25} + 0.01951 \ \Phi_{24} + 0.02439\Phi_{23} + 0.03902\Phi_{22} + 0.04878\Phi_{21} + 0.07805 \ \Phi_{20} + 0.09756\Phi_{19} + 0.1\Phi_{18} + 0.19512\Phi_{17} + 0.2\Phi_{16} + 0.39024\Phi_{15} + 0.4\Phi_{14} + 0.5\Phi_{13} \ + \ 0.78049\Phi_{12} + \Phi_{11} - 1.6\Phi_{10} - 2\Phi_9 \ \ - 1.6\Phi_8 \ + \ 2\Phi_4 \ - \frac{0.0780482565}{35303691240}\Phi_2 - \frac{0.0249397165}{282429529920}\Phi_1,$

$\chi_{656} = 0.00244\Phi_{26} + 0.00305\Phi_{25} + 0.00488\Phi_{24} + 0.00609\Phi_{23} + 0.00976\Phi_{22} + 0.01219\Phi_{21} + 0.01951\Phi_{20} + 0.02439\Phi_{19} + 0.025\Phi_{18} + 0.04878\Phi_{17} + 0.05\Phi_{16} + 0.09756\Phi_{15} + 0.1\Phi_{14} + 0.125\Phi_{13} + 0.2\Phi_{12} + 0.25\Phi_{11} - 0.4\Phi_{10} + 0.5\Phi_9 + 0.8\Phi_8 - 0.5\Phi_6 - 0.5\Phi_5 + \Phi_4 - \frac{0.038169072}{35303691240}\Phi_2 + \frac{0.0019510701}{282429529920}\Phi_1,$

$\chi_{820} = 0.00244\Phi_{26} + 0.00305\Phi_{25} + 0.00488 \ \Phi_{24} + 0.00609\Phi_{23} + 0.00976\Phi_{22} + 0.01219\Phi_{21} + 0.01951 \ \Phi_{20} + 0.02439\Phi_{19} + 0.025\Phi_{18} + 0.04878\Phi_{17} + 0.05\Phi_{16} + 0.09756\Phi_{15} + 0.1\Phi_{14}$

$+ 0.125\Phi_{13} + 0.2\Phi_{12} + 0.25\Phi_{11} - 0.4\Phi_{10} + 0.5\Phi_9 - 0.8\Phi_8 - \Phi_7 + \Phi_4 - \frac{0.074388701}{35303691240}\Phi_2 - \frac{0.0093745997}{282429529920}\Phi_1,$

$\chi_{1312} = 0.00244\Phi_{26} + 0.00305\Phi_{25} + 0.00488\,\Phi_{24} + 0.00609\Phi_{23} + 0.00976\Phi_{22} + 0.01219\Phi_{21} + 0.01951\,\Phi_{20} + 0.02439\Phi_{19} + 0.025\Phi_{18} + 0.04878\Phi_{17} + 0.05\Phi_{16} + 0.09756\Phi_{15} + 0.1\Phi_{14} + 0.125\Phi_{13} + 0.2\Phi_{12} + 0.25\Phi_{11} - 0.4\Phi_{10} + 0.5\Phi_9 + 0.8\Phi_8 + \Phi_7 - \Phi_6 - \Phi_5 + \Phi_4 - \frac{0.0323154134}{35303691240}\Phi_2 - \frac{0.0066311122}{282429529920}\Phi_1,$

$\chi_{1640} = 0.00061\Phi_{26} + 0.00076\Phi_{25} + 0.00122\Phi_{24} + 0.00152\Phi_{23} + 0.00244\Phi_{22} + 0.00305\Phi_{21} + 0.00488\Phi_{20} + 0.00609\Phi_{19} + 0.00625\Phi_{18} + 0.01219\Phi_{17} + 0.0125\Phi_{16} + 0.02439\Phi_{15} + 0.025\Phi_{14} + 0.03125\Phi_{13} + 0.05\Phi_{12} + 0.0625\Phi_{11} + 0.1\Phi_{10} + 0.125\Phi_9 + 0.2\Phi_8 + 0.25\Phi_7 - 0.5\Phi_6 + 0.5\Phi_4 - \frac{0.000542672}{35303691240}\Phi_2 - \frac{1.527019083}{282429529920}\Phi_1,$

$\theta_1 = 764.02073\Phi_{32} - 1528.04145\Phi_{31} + 722.82353\Phi_{30} - 3510.85714\Phi_{29} - 1536\Phi_{28} + 1536\Phi_{27} + 1536\Phi_4 - 3072\Phi_3 + \frac{344266091142559}{35303691240}\Phi_2 - \frac{215947279818000}{282429529920}\Phi_1,$

$\theta_2 = 764.02073\Phi_{31} + 722.82353\Phi_{30} + 1755.42857\Phi_{29} - 768\Phi_{27} - 768\Phi_4 - \frac{172133045737415}{35303691240}\Phi_2 + \frac{64666288341601}{282429529920}\Phi_1,$

$\theta_{17} = -47.75129\Phi_{32} + 95.50259\Phi_{31} - 219.42857\Phi_{29} + 96\Phi_{28} - 96\Phi_{27} + 96\Phi_4 - 192\Phi_3 + \frac{15500650468781}{35303691240}\Phi_2 + \frac{22.3907045099}{282429529920}\Phi_1,$

$\theta_{34} = -23.87565\Phi_{32} - 47.75129\Phi_{31} - 135.52941\Phi_{30} + 96\Phi_{28} - 96\Phi_{27} - 48\Phi_4 + \frac{49.6531614244}{35303691240}\Phi_2 + \frac{7.5525675071}{282429529920}\Phi_1,$

$\theta_{193} = 0.33161\Phi_{32} - 0.66321\Phi_{31} - 3.76471\Phi_{30} + 18.28571\Phi_{29} + 8\Phi_{28} - 8\Phi_{27} + 8\Phi_4 - 16\Phi_3 - \frac{42.684534237}{35303691240}\Phi_2 + \frac{2.9716547752}{282429529920}\Phi_1,$

$\theta_{386} = -0.16580\Phi_{32} - 0.33161\Phi_{31} - 1.88235\Phi_{30} - 9.14286\Phi_{29} - 4\Phi_{27} - 4\Phi_4 - \frac{22.3566760927}{35303691240}\Phi_2 + \frac{1.1822713191}{282429529920}\Phi_1,$

$\zeta = 0.00061\Phi_{26} + 0.00076\Phi_{25} + 0.00122\Phi_{24} + 0.00152\Phi_{23} + 0.00243\Phi_{22} + 0.00305\Phi_{21} + 0.00488\Phi_{20} + 0.00609\Phi_{19} + 0.00625\Phi_{18} + 0.01219\Phi_{17} + 0.125\Phi_{16} - 0.02439\Phi_{15} + 0.025\Phi_{14} + 0.03125\Phi_{13} + 0.05\Phi_{12} + 0.0625\Phi_{11} + 0.1\Phi_{10} + 0.125\Phi_9 - 0.2\Phi_8 + 0.25\Phi_7 + 0.5\Phi_6 - \Phi_5 + 0.5\Phi_4 + \frac{0.0094524031}{35303691240}\Phi_2 - \frac{0.0046098219}{282429529920}\Phi_1,$

$\eta = -0.00518\Phi_{32} + 0.01036\Phi_{31} - 0.05882\Phi_{30} + 0.28571\Phi_{29} - 2\Phi_{28} + 2\Phi_{27} + 0.5\Phi_4 - \Phi_3 - \frac{0.7903068335}{35303691240}\Phi_2 + \frac{0.0683878186}{282429529920}\Phi_1.$

Therefore $\mathcal{A}(\mathcal{SL}(2,3^8)) = 282429529920\chi_1$.

4. Conclusion

From our results we get that the Artin indicator equal to the order of the group $\mathcal{SL}(2,3^8)$ after we compute the rational valued characters of the rational representations written as a linear combination of the induced characters for the groups $\mathcal{SL}(2,3^8)$.

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