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Study of Telegraph Equation via He-Fractional Laplace Homotopy Perturbation Technique

Ali Moazzam*
University of Agriculture, Faisalabad, Pakistan

Ayza Anjum
University of Agriculture, Faisalabad, Pakistan

Nimra Saleem
University of Agriculture Faisalabad, Pakistan

Emad A. Kuffi
Al-Qadisiyah University, College of Engineering, Iraq.

***Corresponding Author:** alimoazzam7309723@gmail.com

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Abstract

A new technique to study the telegraph equation, mostly familiar as damped wave equation is introduced in this study. This phenomenon is mostly rising in electromagnetic influences and production of electric signals. The proposed technique called as He-Fractional Laplace technique with help of Homotopy perturbation is utilized to found the exact and nearly approximated results of differential model and numerical example of telegraph equation or damped wave equation in this article. The most unique term of this technique is that, there is no worry to find the next iteration by integration in recurrence relation. As fractional Laplace integral transformation has some limitations in non-linear terms, to get the result of nonlinear term in this differential mode, He polynomials via homotopy techniques of iteration is proposed to find the result of the computation assignment. The obtained result by this proposed technique directed that this technique is quite ease to apply and convergent rapidly to exact solutions. Numerous examples are described to determine the stability and accuracy of the proposed technique with the graphical explanation.

Keywords: Fractional Laplace method, He's Polynomials, Homotopy Perturbation, Approximated solutions, Telegraph Equation.

1. Introduction

Integral transformations are mathematical operations that transform a complex function in a function space into a simpler function in a transformed space. This transformed function can be easily characterized and manipulated through integration in the transformed function space. The inverse transform technique is often used to convert the transformed function back into its original space.

Although the information on this topic is abundant, the derivation of various research results is only sometimes reliable. The outcomes often show contrasts depending on the material utilized and the analysis area. Some related studies are assessed here.

Many mathematicians have presented the theories of Fourier and Laplace transformations theories, but no one has yet compared k-Fourier and k-Laplace transforms. Ongoing work on modern integral transformations, such as Laplace, Fourier, Mahgoub, Mohand, and Aboodh transformations, is still in progress. The authors of some comparative studies have proven that these transformations are integral to solving many advanced problems in engineering and sciences. Unstable responses, bifurcation situations, chaos, and various other complex effects of vibration typically describe nonlinear vibrational phenomena. It is essential to understand the vibrational behavior to comprehend nonlinear vibration systems better. Factors contributing to vibrational phenomena include nonlinear damping, elastic deformation, and electrical fields.

To proceed, we must consider the following variables:

I Present the current

u Indicate the potential

L Present the inductance

C Shows the Capacitance

G Indicate the conductance leakage

Law of physics which described the relationship between the changes of potential and current in transmission line can be described by the partial difference equation of first order as,

$$Cu_{\xi} + I_{\mu} + Gu = 0 \tag{1}$$

$$LI_{\xi} + u_{\mu} + R_{I} = 0 \tag{2}$$

The above relation is known as Telegraphic equation.

We can get 2nd order differential equation by differentiate (1) with respect to "μ" and (2) with respect to "ξ" and after resolving them, we have

$$u_{\mu\mu} + L\left[-Gu_{\xi} - Cu_{\xi}\right] + R\left[-Gu_{\xi} - Cu_{\xi}\right] = 0$$

This can also write as,

$$\gamma^2 u_{\mu\mu} = u_{\xi\xi} + \alpha\beta u_{\xi} + (\alpha+\beta)u_{\xi} \tag{3}$$

Where $\alpha = \dfrac{R}{L}, \gamma = \sqrt{\dfrac{1}{LC}}, \beta = \dfrac{G}{C}$ this is the telegraph equation which take place in electrometric waves.

Many works on the numerical solution of differential 2nd order hyperbolic equations have been done in previous years. An algorithm of numerical solution of a telegraphic equation is described by [2]. In [3], the author formulated a scheming couple for Haar wavelets and finite differences to solve telegraphic hyperbolic equality with some variable coefficients. The solution of the telegraphic equation is also explained in [4,5] for the nonlinear phenomenon. Kamal integral technique was used with delay differential equations in [6]. The differential-integral equations in [7] were resolved using modified Laplace series results via the Adomian polynomial decomposition method (LADM). The conventional Adomian polynomials were utilized to get around the non-linearity. To find out the numerical approximated results of the system of partial on linear differential equations in [8], practiced the Laplace decomposition approach and the pade approximation. It also modified the Laplace decomposition method used in [9] for differential equalities of the Emden-Lane type. The first and second kinds of non-linear Volterra-Fredholm integro differential equations were solved analytically by the author [10] using a collective form of the modified Laplace Adomian substitution and Laplace Adomian polynomial decomposition method (LADM). There are many applications for partial differential equations in the sciences. There are many different ways to solve partial differential equations. Engineering and scientific sectors frequently employ used Laplace integral techniques. Numerous writers [11-14] have discussed how integral transformations can resolve improper integrals that include error functions. [15] Used the Kamal integral technique to find the answers to Abel's equation. [16] offered a Kamal integral technique-based solution to the error function.

In [17], the authors described the substitution method of Laplace transforms to solve linear partial differential equations with more than two independent variables. The influence of Bio-convection and activation energy on the Maxwell equation on nano-fluid has been explained by [18] with the help of the Matlab program. An analytical solution of the advection-diffusion equation in one-dimensional with a semi-infinite medium has been elaborated by [19] with the help of the Laplace transformation technique. In 2021, researchers in [20,21] introduced a new transformation to solve differential equations with trigonometric coefficients. The application of Kamal transformation in thermal engineering has been shown in [22], and the solution to the temperature problem and some of its applications are shown in the article. In [23], a new SEE transformation has been shown, which is quite helpful in solving differential equations and systems of differential equations. The solution of differential equations of moment Pareto distribution with logarithmic coefficients has been shown in [24] with the help of the Al-Zughair transformation. A new transformation in the logarithmic kernel has been shown in the article [25]; this transformation helps solve differential equations with logarithmic coefficients and also ordinary differential equations.
The main concept of the variation iterative method (VIM) gave by [26], but finding the Lagrange multiplier (LM) was challenging. After that, VIM introduced a methodology as He’s polynomial mentioned in [27-29], and many applications of this are seen in [30-32] to get solutions to non-linear problems. Approximated results obtained by VIM converge rapidly to the exact solution. Also, the solution of the telegraphic equation by VIM is shown in [33]. Many researchers find the approximate solution of many non-linear differential equations via homotopy perturbation shown in [34-39] and in [40,41], authors used the combined technique of Laplace and homotopy perturbation to find the Lagrange multiplier. Sometimes, finding the value of LM is difficult due to some limitations of existing techniques. To overcome all hurdles, we introduced a new technique

to find the analytical results of the telegraphic mathematical model. In this article, we will introduce He-Fraction Laplace integral transformation and its applications in resolving the telegraph model or damped wave mathematical model. Fractional Laplace transformation can be defined as below.

Definition:[42] Integral Fractional Laplace transformation of the function $f(\xi)$ for all $\xi \geq 0$ is defined as

$$L_\alpha\left\{f(\xi)\right\} = \int_0^\infty e^{\xi v^{\frac{1}{\alpha}}} f(\xi) d\xi \tag{4}$$

Where α is real numbers and v is transformed variable. By choosing the values of parameter $\alpha = 1$ as arbitrary real number, we can get the Laplace transformations as Laplace.

2. Methodology

a Decomposition Homotopy Perturbation Technique of Nonlinear Equations

For the implementation of homotopy perturbation method, let assume a nonlinear equation as,

$$R(\varpi) = c + S(\varpi) \tag{5}$$

In above S, R and c described the nonlinear term, linear term and source value respectively. By using the homotopy technique we have

$$H(\phi,\psi) : \mathbf{R} \times [0,1] \to \mathbf{R}$$

As

$$H(\phi,\psi) = (1-\psi)\left[R(\varpi) - R(\varpi_0)\right] + \psi\left[R(\varpi) - S(\varpi) - c\right]$$

$$= R(\varpi) - R(\varpi_0) + \psi R(\varpi) - \psi S(\varpi) - \psi c = 0 \tag{6}$$

$$= R(\varpi) - c - \psi S(\varpi) = 0$$

Here, $R(\varpi_0) = c$

$$\Rightarrow \quad R(\varpi) = c + \psi S(\varpi)$$

In this technique, solution ϕ can be shown as,

$$\phi = \lim_{\psi \to 0}\left(\phi_0 + \psi\phi_1 + \psi^2\phi_2 + \psi^3\phi_3 + \ldots\right), \tag{7}$$

$$= \left(\phi_0 + \phi_1 + \phi_2 + \phi_3 + \ldots\right),$$

$$= \sum_0^\infty \phi_i \tag{8}$$

In this $\psi \in [0,1]$ and ϕ_0 is the first approximation, by using (6) and (7)

$$\phi_0+\psi\phi_1+\psi^2\phi_2+\psi^3\phi_3+\ldots=c+\psi\left\{\begin{array}{l}S(\phi_0)+\psi(\phi_{01}+\psi\phi_{02}+\ldots)\\ S^{'}(\phi_0)+\dfrac{\psi^2}{2}(\phi_{01}+\psi\phi_{02}+\ldots)\\ S^{"}(\phi_0)+\ldots\end{array}\right\}$$

By equating the powers of ψ, one has

$$\psi^0:\phi_0=c,$$

$$\psi^1:\phi_1=S(\phi_0),$$

$$\psi^2:\phi_2=(S)^{'}$$

$$\psi^3:\phi_3=(S)^{"}$$

$$\vdots$$

So equation (8) can be expressed as,

$$\phi=\phi_0+\phi_1+\phi_2+\phi_3+\ldots$$

2.2. He–Fractional-Laplace Technique

To find the He-Fractional Laplace technique, first we need to take Fractional Laplace integral transformation of equation (5)

$$\mathrm{L}_\alpha\left[R(\varpi(\mu))-S(\varpi(\mu))-c\right]=0 \tag{9}$$

It implies,

$$\lambda\left\{c+\mathrm{L}_\alpha\left[R(\varpi(\mu))\right]-\mathrm{L}_\alpha\left[S(\varpi(\mu))\right]\right\}=0$$

Reccurrence relation can be expressed as,

$$\varpi_{n+1}\left(v^{\frac{1}{\alpha}}\right)=\varpi_n\left(v^{\frac{1}{\alpha}}\right)+\lambda\left\{\mathrm{L}_\alpha\left[R(\varpi(\mu))\right]-\mathrm{L}_\alpha\left[S(\varpi(\mu))\right]+c\right\} \tag{10}$$

we may use optimal conditions, to identify the Lagrange multiplier $\lambda\left(v^{\frac{1}{\alpha}}\right)$

$$\frac{\delta\varpi_{n+1}\left(v^{\frac{1}{\alpha}}\right)}{\delta\varpi_n\left(v^{\frac{1}{\alpha}}\right)}=0$$

by applying the inverse Fractional-Laplace integral transformation of eq. (10) we get,

$$\varpi_{n+1}\left(v^{\frac{1}{\alpha}}\right)=\varpi_{n}\left(v^{\frac{1}{\alpha}}\right)+L_{\alpha}^{-1}\left[\lambda\left\{L_{\alpha}\left[R\left(\varpi\left(\mu\right)\right)\right]-L_{\alpha}\left[S\left(\varpi\left(\mu\right)\right)\right]+c\right\}\right] \tag{11}$$

At the end, Homotopy perturbation technique has used to find the approximation series results by equating the degrees of ψ .

3. Application

Analytical solution of telegraph equation has been found in this section with the help of He-Fractional Laplace technique. Proposed method shows the significant and novel behavior in results obtained. Illustrated numerical examples provides the existence and efficiency of proposed technique.

Example 1:

Let assume the proceeding telegraph equation

$$\varpi_{\mu\mu}=\varpi_{\xi\xi}+\varpi_{\xi}+\varpi \tag{12}$$

With given initial and boundary values respectively,

$$\varpi\left(\mu,0\right)=e^{\mu},\qquad \varpi_{\xi}\left(\mu,0\right)=-2e^{\mu} \tag{13}$$

$$\varpi\left(0,\xi\right)=e^{-2\xi},\qquad \varpi_{\xi}\left(0,\xi\right)=e^{-2\xi} \tag{14}$$

By using the Fractional-Laplace integral technique on equation (12)

$$L_{\alpha}\left[\frac{\partial^{2}\varpi}{\partial\xi^{2}}+\frac{\partial\varpi}{\partial\xi}-\varpi-\frac{\partial^{2}\varpi}{\partial\mu^{2}}\right]=0$$

Multiply the above mentioned equation by $\lambda_1\left(v^{\frac{1}{\alpha}}\right)$, we get

$$\lambda_1 L_{\alpha}\left[\frac{\partial^{2}\varpi}{\partial\xi^{2}}+\frac{\partial\varpi}{\partial\xi}-\varpi-\frac{\partial^{2}\varpi}{\partial\mu^{2}}\right]=0$$

Recurrence relation can be expressed as,

$$\varpi_{n+1}\left(\mu,v^{\frac{1}{\alpha}}\right)=\varpi_{n}\left(\mu,v^{\frac{1}{\alpha}}\right)+\lambda_1 L_{\alpha}\left[\frac{\partial^{2}\varpi}{\partial\xi^{2}}+\frac{\partial\varpi}{\partial\xi}-\varpi-\frac{\partial^{2}\varpi}{\partial\mu^{2}}\right] \tag{15}$$

By using the variational parameter in eq. (15)

$$\delta\varpi_{n+1}\left(\mu,v^{\frac{1}{\alpha}}\right)=\delta\varpi_{n}\left(\mu,v^{\frac{1}{\alpha}}\right)+\lambda_1 L_{\alpha}\delta\left[\frac{\partial^{2}\varpi}{\partial\xi^{2}}+\frac{\partial\varpi}{\partial\xi}-\varpi-\frac{\partial^{2}\varpi}{\partial\mu^{2}}\right]$$

$$\delta\varpi_{n+1}\left(\mu,v^{\frac{1}{\alpha}}\right)=\delta\varpi_{n}\left(\mu,v^{\frac{1}{\alpha}}\right)+\lambda_1\delta\left[\left\{\left(v^{\frac{1}{\alpha}}\right)^{2}\varpi_{n}\left(\mu,v^{\frac{1}{\alpha}}\right)-v^{\frac{1}{\alpha}}\varpi_{n}\left(\mu,0\right)\right\}+L_{\alpha}\left\{\frac{\partial\varpi}{\partial\xi}-\varpi-\frac{\partial^{2}\varpi}{\partial\mu^{2}}\right\}\right],$$

$$\delta\varpi_{n+1}\left(\mu,v^{\frac{1}{\alpha}}\right)=\delta\varpi_{n}\left(\mu,v^{\frac{1}{\alpha}}\right)+\left(v^{\frac{1}{\alpha}}\right)^{2}\lambda_{1}\varpi_{n}\left(\mu,v^{\frac{1}{\alpha}}\right)$$

It will turn into,

$$\lambda_{1}\left(v^{\frac{1}{\alpha}}\right)=-\frac{1}{\left(v^{\frac{1}{\alpha}}\right)^{2}}$$

Here ω_n is restricted as $\delta\omega_n=0$ and

$$\frac{\delta\varpi_{n+1}\left(\mu,v^{\frac{1}{\alpha}}\right)}{\delta\varpi_{n}\left(\mu,v^{\frac{1}{\alpha}}\right)}=0$$

By putting the value of $\lambda_1(s)$ in eq. (15)

$$\delta\varpi_{n+1}\left(\mu,v^{\frac{1}{\alpha}}\right)=\delta\varpi_{n}\left(\mu,v^{\frac{1}{\alpha}}\right)-\frac{1}{\left(v^{\frac{1}{\alpha}}\right)^{2}}\mathrm{L}_{\alpha}\left[\frac{\partial^{2}\varpi_{n}}{\partial\xi^{2}}+\frac{\partial\varpi_{n}}{\partial\xi}-\varpi_{n}-\frac{\partial^{2}\varpi_{n}}{\partial\mu^{2}}\right]$$

By using the inverse Fractional-Laplace in above mentioned equation

$$\varpi_{n+1}(\mu,\xi)=\varpi_{n}(\mu,\xi)-\mathrm{L}_{\alpha}^{-1}\left[\frac{1}{\left(v^{\frac{1}{\alpha}}\right)^{2}}\mathrm{L}_{\alpha}\left[\frac{\partial^{2}\varpi_{n}}{\partial\xi^{2}}+\frac{\partial\varpi_{n}}{\partial\xi}-\varpi_{n}-\frac{\partial^{2}\varpi_{n}}{\partial\mu^{2}}\right]\right]$$

By the help of He’s polynomials, above equation turns into

$$\varpi_{0}+\psi\varpi_{1}+\psi^{2}\varpi_{2}+\psi^{4}\varpi_{4}+\cdots=\varpi_{n}(\mu,\xi)-\psi\mathrm{L}_{\alpha}^{-1}\left[\frac{1}{\left(v^{\frac{1}{\alpha}}\right)^{2}}\mathrm{L}_{\alpha}\left[\begin{array}{l}\frac{\partial^{2}\varpi_{n}}{\partial\xi^{2}}+\frac{\partial\varpi_{n}}{\partial\xi}-\\ \varpi_{n}-\frac{\partial^{2}\varpi_{n}}{\partial\mu^{2}}\end{array}\right]\right]$$

It can also written as,

$$\varpi_{0}+\psi\varpi_{1}+\psi^{2}\varpi_{2}+\psi^{4}\varpi_{4}+\cdots=\varpi_{n}(\mu,\xi)-\psi\mathrm{L}_{\alpha}^{-1}\left[\frac{1}{\left(v^{\frac{1}{\alpha}}\right)^{2}}\mathrm{L}_{\alpha}\left[\frac{\partial\varpi_{n}}{\partial\xi}-\varpi_{n}-\frac{\partial^{2}\varpi_{n}}{\partial\mu^{2}}\right]\right],$$

$$=\varpi_n(\mu,\xi)-\psi \mathrm{L}_\alpha^{-1}\left[\frac{1}{\left(v^{\frac{1}{\alpha}}\right)^2}\mathrm{L}_\alpha\left\{\begin{array}{l}\left(\frac{\partial \varpi_0}{\partial \xi}-\varpi_0-\frac{\partial^2 \varpi_0}{\partial \mu^2}\right)+\psi\left(\frac{\partial \varpi_1}{\partial \xi}-\varpi_1-\frac{\partial^2 \varpi_1}{\partial \mu^2}\right)+ \\ \psi^2\left(\frac{\partial \varpi_2}{\partial \xi}-\varpi_2-\frac{\partial^2 \varpi_2}{\partial \mu^2}\right)+\psi^3\left(\frac{\partial \varpi_3}{\partial \xi}-\varpi_3-\frac{\partial^2 \varpi_3}{\partial \mu^2}\right)\end{array}\right\}\right],$$

By equating highest powers of Ψ

$$\psi^0:\varpi_0=\varpi_0(\mu,\xi)+\xi\varpi_0(\mu,\xi),$$

$$\psi^1:\varpi_1=-\mathrm{L}_\alpha^{-1}\left[\frac{1}{\left(v^{\frac{1}{\alpha}}\right)^2}\mathrm{L}_\alpha\left(\frac{\partial \varpi_1}{\partial \xi}-\varpi_1-\frac{\partial^2 \varpi_1}{\partial \mu^2}\right)\right]$$

$$\psi^2:\varpi_2=-\mathrm{L}_\alpha^{-1}\left[\frac{1}{\left(v^{\frac{1}{\alpha}}\right)^2}\mathrm{L}_\alpha\left(\frac{\partial \varpi_2}{\partial \xi}-\varpi_2-\frac{\partial^2 \varpi_2}{\partial \mu^2}\right)\right]$$

$$\psi^3:\varpi_3=-\mathrm{L}_\alpha^{-1}\left[\frac{1}{\left(v^{\frac{1}{\alpha}}\right)^2}\mathrm{L}_\alpha\left(\frac{\partial \varpi_3}{\partial \xi}-\varpi_3-\frac{\partial^2 \varpi_3}{\partial \mu^2}\right)\right]$$

$$\psi^4:\varpi_4=-\mathrm{L}_\alpha^{-1}\left[\frac{1}{\left(v^{\frac{1}{\alpha}}\right)^2}\mathrm{L}_\alpha\left(\frac{\partial \varpi_4}{\partial \xi}-\varpi_4-\frac{\partial^2 \varpi_4}{\partial \mu^2}\right)\right]$$

$$\vdots$$

so, we get

$$\varpi_0=(1-2\xi)e^{\mu}$$

$$\varpi_1=\left(2\xi^2-\frac{2}{3}\xi^3\right)e^{\mu}$$

$$\varpi_2=\left(-\frac{2}{3}\xi^3+\frac{1}{2}\xi^4-\frac{1}{15}\xi^5\right)e^{\mu}$$

$$\varpi_3 = \left(\frac{1}{6}\xi^4 - \frac{1}{6}\xi^5 + \frac{2}{45}\xi^6 - \frac{1}{315}\xi^7\right)e^{\mu}$$

$$\varpi_4 = \left(-\frac{1}{30}\xi^5 + \frac{7}{180}\xi^6 - \frac{1}{70}\xi^7 + \frac{1}{504}\xi^8 - \frac{1}{11340}\xi^9\right)e^{\mu}$$

$$\vdots$$

Hence this result can be expressed as,

$$\varpi_n(\mu,\xi) = \varpi_0 + \varpi_1 + \varpi_2 + \ldots + \varpi_n$$

$$\varpi_n(\mu,\xi) = e^{\mu}\left(1 - 2\xi + 2\xi^2 - \frac{4}{3}\xi^3 + \frac{2}{3}\xi^4 - \frac{4}{15}\xi^5 + \ldots\right) \tag{16}$$

Result is rapidly convergent to below equation,

$$\varpi_n(\mu,\xi) = e^{\mu-2\xi} \tag{17}$$

Equation (17) provides the exact solution and (16) provides the He-Fractional Fractional-Laplace approximated solution.

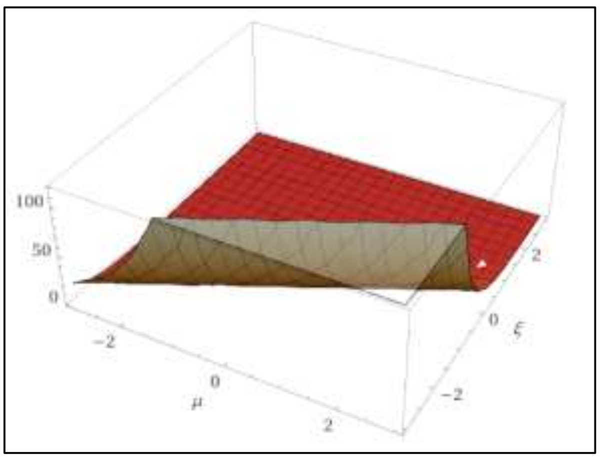

Figure 1: The given diagram explain the exact solution of Example 1.

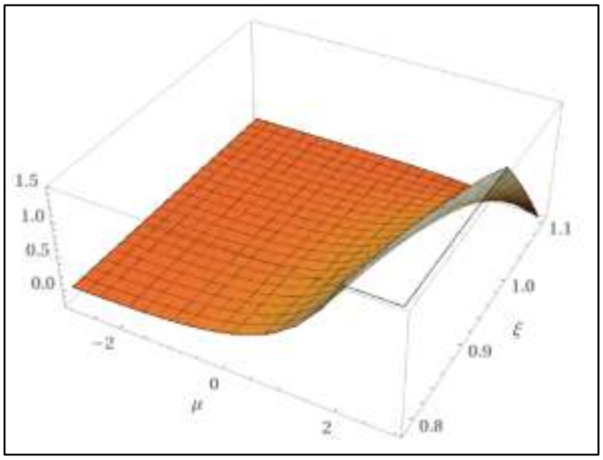

Figure 2: The given diagram explain the He-Fractional Fractional-Laplace solution of Example 1.

Example 2:

Let assume the proceeding telegraph equation,

$$\varpi_{\mu\mu} = \varpi_{\xi\xi} + 4\varpi_{\xi} + 4\varpi \tag{18}$$

With given initial and boundary values respectively,

$$\varpi(\mu,0) = 1 + e^{2\mu}, \qquad \varpi_{\xi}(\mu,0) = -2 \tag{19}$$

$$\varpi(0,\xi) = 1 + e^{-2\xi}, \qquad \varpi_{\xi}(0,\xi) = 2 \tag{20}$$

By using the Fractional-Laplace integral technique on equation (18)

$$\mathrm{L}_{\alpha}\left[\frac{\partial^2 \varpi}{\partial \xi^2} + 4\frac{\partial \varpi}{\partial \xi} + 4\varpi - 4\frac{\partial^2 \varpi}{\partial \mu^2}\right] = 0$$

Multiply the above mentioned equation by $\lambda_2\left(v^{\frac{1}{\alpha}}\right)$

$$\lambda_2 \mathrm{L}_{\alpha}\left[\frac{\partial^2 \varpi}{\partial \xi^2} + 4\frac{\partial \varpi}{\partial \xi} + 4\varpi - 4\frac{\partial^2 \varpi}{\partial \mu^2}\right] = 0$$

Recurrence relation can be expressed as,

$$\varpi_{n+1}\left(\mu, v^{\frac{1}{\alpha}}\right) = \varpi_n\left(\mu, v^{\frac{1}{\alpha}}\right) + \lambda_2 \mathrm{L}_{\alpha}\left[\frac{\partial^2 \varpi_n}{\partial \xi^2} + 4\frac{\partial \varpi_n}{\partial \xi} + 4\varpi_n - 4\frac{\partial^2 \varpi_n}{\partial \mu^2}\right] \tag{21}$$

By using the variation in equation (21)

$$\delta\varpi_{n+1}\left(\mu, v^{\frac{1}{\alpha}}\right) = \delta\varpi_n\left(\mu, v^{\frac{1}{\alpha}}\right) + \lambda_2 \mathrm{L}_{\alpha}\delta\left[\frac{\partial^2 \varpi_n}{\partial \xi^2} + 4\frac{\partial \varpi_n}{\partial \xi} + 4\varpi_n - 4\frac{\partial^2 \varpi_n}{\partial \mu^2}\right],$$

$$\delta\varpi_{n+1}\left(\mu, v^{\frac{1}{\alpha}}\right) = \delta\varpi_n\left(\mu, v^{\frac{1}{\alpha}}\right) + \left(v^{\frac{1}{\alpha}}\right)^2 \lambda_2 \delta\varpi_n\left(\mu, v^{\frac{1}{\alpha}}\right)$$

It will turn into,

$$\lambda_2\left(v^{\frac{1}{\alpha}}\right) = -\frac{1}{\left(v^{\frac{1}{\alpha}}\right)^2}$$

Here ω_n is restricted as $\delta\omega_n = 0$ and

$$\frac{\delta\varpi_{n+1}\left(\mu, v^{\frac{1}{\alpha}}\right)}{\delta\varpi_n\left(\mu, v^{\frac{1}{\alpha}}\right)} = 0$$

By putting the value of $\lambda_2\left(v^{\frac{1}{\alpha}}\right)$ in equation (21)

$$\varpi_{n+1}\left(\mu, v^{\frac{1}{\alpha}}\right)=\varpi_n\left(\mu, v^{\frac{1}{\alpha}}\right)-\frac{1}{\left(v^{\frac{1}{\alpha}}\right)^2}\mathrm{L}_\alpha\left[\frac{\partial^2\varpi_n}{\partial\xi^2}+4\frac{\partial\varpi_n}{\partial\xi}+4\varpi_n-4\frac{\partial^2\varpi_n}{\partial\mu^2}\right]$$

By using the inverse Fractional-Laplace in above mentioned equation

$$\varpi_{n+1}\left(\mu, v^{\frac{1}{\alpha}}\right)=\varpi_n(\mu,\xi)-\mathrm{L}_\alpha^{-1}\left[\frac{1}{\left(v^{\frac{1}{\alpha}}\right)^2}\mathrm{L}_\alpha\left\{\frac{\partial^2\varpi_n}{\partial\xi^2}+4\frac{\partial\varpi_n}{\partial\xi}+4\varpi_n-4\frac{\partial^2\varpi_n}{\partial\mu^2}\right\}\right]$$

By the help of He's polynomials, above equation turns into

$$\varpi_0+\psi\varpi_1+\psi^2\varpi_2+\psi^3\varpi_3+\psi^4\varpi_4+\cdots=\varpi_n(\mu,\xi)-\psi\mathrm{L}_\alpha^{-1}\left[\frac{1}{\left(v^{\frac{1}{\alpha}}\right)^2}\mathrm{L}_\alpha\left\{\begin{matrix}\frac{\partial^2\varpi_n}{\partial\xi^2}+4\frac{\partial\varpi_n}{\partial\xi}+\\4\varpi_n-4\frac{\partial^2\varpi_n}{\partial\mu^2}\end{matrix}\right\}\right]$$

It can also written as,

$$\varpi_0+\psi\varpi_1+\psi^2\varpi_2+\psi^3\varpi_3+\psi^4\varpi_4+\cdots=\varpi_n(\mu,\xi)-\psi\mathrm{L}_\alpha^{-1}\left[\frac{1}{\left(v^{\frac{1}{\alpha}}\right)^2}\mathrm{L}_\alpha\left\{\begin{matrix}4\frac{\partial\varpi_n}{\partial\xi}+4\varpi_n-\\4\frac{\partial^2\varpi_n}{\partial\mu^2}\end{matrix}\right\}\right]$$

$$=\varpi_n(\mu,\xi)-\psi\mathrm{L}_\alpha^{-1}\left[\frac{1}{\left(v^{\frac{1}{\alpha}}\right)^2}\mathrm{L}_\alpha\left\{\begin{matrix}\left(4\frac{\partial\varpi_n}{\partial\xi}+4\varpi_n-4\frac{\partial^2\varpi_n}{\partial\mu^2}\right)+\psi\left(4\frac{\partial\varpi_n}{\partial\xi}+4\varpi_n-4\frac{\partial^2\varpi_n}{\partial\mu^2}\right)\\+\psi^2\left(4\frac{\partial\varpi_n}{\partial\xi}+4\varpi_n-4\frac{\partial^2\varpi_n}{\partial\mu^2}\right)+\psi^3\left(4\frac{\partial\varpi_n}{\partial\xi}+4\varpi_n-4\frac{\partial^2\varpi_n}{\partial\mu^2}\right)\\+\cdots\end{matrix}\right\}\right]$$

By equating highest powers of ψ

$$\psi^0:\varpi_0=\varpi_0(\mu,\xi)+\xi\varpi_{0\xi}(\mu,\xi)$$

$$\psi^1 : \varpi_1 = -L_\alpha^{-1}\left[\frac{1}{\left(v^{\frac{1}{\alpha}}\right)^2} L_\alpha \left\{4\frac{\partial \varpi_0}{\partial \xi} + 4\varpi_0 - 4\frac{\partial^2 \varpi_0}{\partial \mu^2}\right\}\right]$$

$$\psi^2 : \varpi_2 = -L_\alpha^{-1}\left[\frac{1}{\left(v^{\frac{1}{\alpha}}\right)^2} L_\alpha \left\{4\frac{\partial \varpi_1}{\partial \xi} + 4\varpi_1 - 4\frac{\partial^2 \varpi_1}{\partial \mu^2}\right\}\right]$$

$$\psi^3 : \varpi_3 = -L_\alpha^{-1}\left[\frac{1}{\left(v^{\frac{1}{\alpha}}\right)^2} L_\alpha \left\{4\frac{\partial \varpi_2}{\partial \xi} + 4\varpi_2 - 4\frac{\partial^2 \varpi_2}{\partial \mu^2}\right\}\right]$$

$$\psi^4 : \varpi_4 = -L_\alpha^{-1}\left[\frac{1}{\left(v^{\frac{1}{\alpha}}\right)^2} L_\alpha \left\{4\frac{\partial \varpi_3}{\partial \xi} + 4\varpi_3 - 4\frac{\partial^2 \varpi_3}{\partial \mu^2}\right\}\right]$$

$$\vdots$$

so, we get

$$\varpi_0 = 1 + e^{2\mu} - 2\xi,$$

$$\varpi_1 = 2\xi^2 + \frac{4}{3}\xi^3,$$

$$\varpi_2 = -\frac{8}{3}\xi^3 - 2\xi^4 - \frac{4}{15}\xi^5,$$

$$\varpi_3 = \frac{8}{3}\xi^4 + \frac{32}{15}\xi^5 + \frac{4}{9}\xi^6 + \frac{8}{315}\xi^7,$$

$$\varpi_4 = -\frac{32}{15}\xi^5 - \frac{16}{9}\xi^6 - \frac{16}{36}\xi^7 - \frac{2}{45}\xi^8 - \frac{4}{2835}\xi^9,$$

$$\vdots$$

So by using the series terms, solution can be written as,

$$\varpi_n(\mu,\xi) = \varpi_0 + \varpi_1 + \varpi_2 + \varpi_3 + \varpi_4 + \cdots,$$

$$\varpi_n(\mu,\xi)=e^{2\eta}+1-2\xi+2\xi^2-\frac{4}{3}\xi^3+\frac{2}{3}\xi^4-\frac{4}{15}\xi^5+\cdots \qquad (22)$$

Result is rapidly convergent to below equation,

$$\varpi_n(\mu,\xi)=e^{2\mu}+e^{-2\xi} \qquad (23)$$

Equation (23) provides the exact solution and (22) provides the He-Fractional Fractional-Laplace approximated solution.

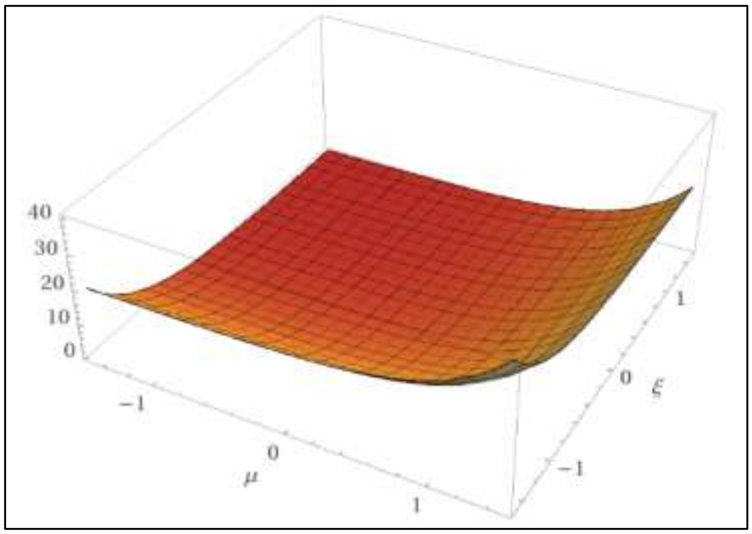

Figure 3: The given diagram explain the exact solution of Example 2.

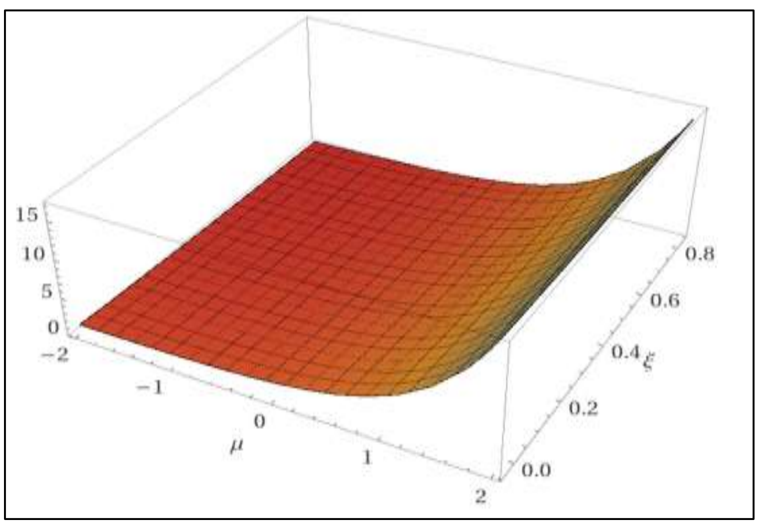

Figure 4: The given diagram explain the He-Fractional Fractional-Laplace solution of Example 2.

4. Conclusion

In this research, He–Fractional-Laplace technique has been shown to solve non-linear and linear partial differential telegraph equations which elaborate the behavior of current and voltage in an electric transmitted line with different distance and time. It is shown that the proposed technique provides the easiest steps for Lagrange identifier other than already existing techniques like VIM and ADM (Adomian Decomposition Method). I t is concluded from the existing results in this study that He–Fractional Laplace technique is quite suitable and reliable for resolving initial value problems as well boundary problems in applied sciences. On the behalf of above computation, following results are drawn.

1. The results obtained by this technique shows the efficiency of He-Fraction Laplace homotopy perturbation techniques.
2. The proposed method has less error, avoid the assumptions and discretization of variables.
3. Partial differential linear and nonlinear equations can be solved by the proposed method and the exact solution can get after some iteration.
4. This proposed has direct command for LM in both case of linear and nonlinear phenomenon.
5. From the above all discussions, it is cleared that the proposed technique is not only restricted to solve nonlinear vibration phenomenon. It is valid form also other nonlinear and linear problems.

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