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Centralizer on Lie-ideal of Semi-prime Inverse Semi-ring

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Abstract

The summary purpose of this work: We extending certain results on α-centralizer of inverse semiring under specific conditions, achieve new results on lie ideal of inverse semiring with some consequent collieries, generalize assorted α-centralizer for lie ideal of inverse semiring with some collieries, investigate significant theorems on jordan α-centralizer of prime inverse semiring and we extend certain results of $\alpha -$centralizers and jordan $\alpha -$centralizers on lie-ideals of prime semi-rings to prime inverse semi-ring, we generalizing the results of Mary in to α-centralizer on semiring, Also we generalize our results on lie ideals of inverse semiring. We extending the results of Shafiq, Aslam, Javed to $\alpha -$ centralizer of Inverse semiring. $since$ R is left (right) Jordan $\alpha -$ centralizer on V, we get the output R is a left (right) $\alpha -$ centralizer on V. If it where α is an automorphism of V,$R(u) \in V$, for any $u \in V$, and $\alpha\big(Z(V)\big) = Z(V)$. We also get the following output R is a $\alpha -$ centralizer on V.

Keywords: Lie-ideal, prime inverse semi-ring, semi-prime inverse semi-ring, $\alpha -$centralizer, jordan α-centralizer.

1. Introduction

Let M be a non-empty set with binary operation $(\bullet)$ defined on M, then $(M,\bullet)$ is named semi $-$ group iff $k \bullet (s \bullet t) = (k \bullet s) \bullet t$ for any $k, s, t \in M$(1), a semi $-$ group M is named commutative semi $-$ group if $k \bullet s = s \bullet k$, holds $for\ all$ $k, s \in M$ (1), A non $-$ empty set with two $-$ binary operations$(+)$ and $(\bullet)$ is named semi-ring iff the following requirements hold:

i) $(M, +)$ is commutative semi $-$ group.

ii) $(M,\bullet)$ semi $-$ group.

iii) $a \bullet (k + s) = a \bullet k + a \bullet s$ and $(k + s) \bullet a = k \bullet a + s \bullet a$ $for\ all\ a, k, s \in M$ (2), $(M, +)$ is named additive commutative with neutral element 0. (i.e. $for\ all\ k \in M,\ k + 0 = 0 + k = k$) iff $k + s = k + n$ holds for any $k, s \in M$, and $(M,\bullet)$ is a semi $-$ group with zero 0, $i.e., 0.a = a.0 = 0$ for any $a \in M$. A semi $-$ ring $(M, +, \bullet)$ is named commutative iff $k \bullet s = s \bullet k$ holds for any $k, s \in M$ (2), Let $(M, +, \bullet)$ be an additively commutative semi-ring. Then M is named inverse semi-ring, if $(M, +)$ is an inverse semi-group (i.e) for each $k \in M$ there are a unique $k' \in M$ such that, $k = k + k' + k$ and $k' + k + k' = k'$ (2), and is called cancellative semi $-$ ring iff for any $k, s, m \in M$, such that $k + s = k + m$, then $s = m$. A semi-ring M is named prime semi-ring if for any $k, s \in M$, $k\,M\,s = 0$ implies that either $k = 0$ or $s = 0$. A semi $-$ ring M is named a semi-prime if for any $k \in M, k\,M\,k = 0$ mplies that $k = 0$. (3), A semi-ring M is named $q -$ torsion free where $q \neq 0$ is an integer if whenever $qk = 0$ with $k \in M$, then $k = 0$. A commutator $[.,.]$ in inverse semi $-$ rings defines as $[k, s] = ks + ks'\ and, k\ o\ s = ks + ks$ (3). In (4) Albas presented the α $-$ centralizer concept and the Jordan α $-$centralizer concept, which could be a generalization of Jordan centralizer and centralizer and tried beneath particular requirements on a 2 $-$torsion free semi $-$ prime ring, each Jordan α-centralizer is α centralizer, where α could be a surjective homomorphism. Inverse semi-rings considered in different directions by numerous authors, see (5-12). In this work our aim is to consider the results of Majeed and Meften (13) in the inverse semi-ring. In this article, M will represent additive inverse semi-ring that satisfies the requirement that for any $r \in M$, $k + \acute{k}$ is located in the center $Z(M)$ of M .

2. Preliminaries

We recalled the definitions of lie $-$ ideal, square closed Lie $-$ ideal of a semiring M, and some definitions, lemmas that will be used later.

Definition (2.1):(14)

An additive sub semi $-$ group of inverse semi $-$ ring M satisfies $[n, q] = nq + q'k \in V$ for any $k \in V$, $q \in M$, is named a Lie-ideal of M .

Definition (2.2):(14)

Let V be a lie $-$ ideal of a ring, then V is named a squane closed Lie $-$ ideal of M if $k^2 \in V\ for\ all\ k \in V$.

Note that if V is a square closed Lie-idealof M, then $2kq \in V$ for any $k, q \in V$.

Definition (2.3):(2), (15)

Let I be a nonzero ideal of M, the set $Z(I) = \{k \in I, kq = qk,$ for any $q \in I\}$ is named the center of I.

Definition (2.4):(2), (16)

Let $q \in M$, the set $Z(M) = \{k \in M, kq = qk, for\ all\ q \in M\}$ is named the center of the semi $-$ ring M. Clearly that $Z(M)$ is a subsemi $-$ ring of M.

Note that if M is multiplicatively commutative then $Z(M) = M$.

Lemma (2.5):(10), (17)

Let M be an additive inverse semi-ring, for any k, $q \in M, if\ k + q = 0$ then $k = q'$. Note that in general $k + k' \neq 0$, $k + k' = 0$, iff there are some $q \epsilon M$ with $k + q = 0$ [2]

Proposition (2.6):(12),(18)

For any $r, s \in M$, the following are holds:

i. $(k + q)' = k' + q'$
ii. $(k\, q)'' = k'q = kq'$
iii. $k'' = k$
iv. $k'q' = (k'q)' = (kq)'' = kq.$

Lemma (2.7):(12),(19)

Let M be ring and k, $q, w \in M$ then

i. $[k, k] = 0$
ii. $[k + q, w] = [k, w] + [q, w]$
iii. $[kq, w] = k[q, w] + [k, w]q$
iv. $[k, qw] = q[k, w] + [k, q]w.$

Definition (2.8):(15),(20)

Let M be a semi-ring,an additive mapping $R: M \to M$ is nameda $(\alpha, \alpha) -$ derivation $if\ R(kq) = R(k)\alpha(q) + \alpha(k)R(q)$ for any $k, q \in M$, and we say that R is Jordan $(\alpha, \alpha) -$ derivation if $R(k^2) = R(k)\alpha(k) + \alpha(k)R(k)$ for any $k \in M$,where α be additive mapping on M.

Every derivation is $(\alpha, \alpha) -$ derivation is Jordan $(\alpha, \alpha) -$ derivation, but the converse in general is not true.

Definition (2.9):(3),(21)

A left (right)$\alpha -$ centralizer of a semi-ring M is an "additive mapping" $R: M \to M$ which satisfies $R(kq) + R(k)\alpha(q)' = 0, (R(kq) + \alpha(k)'R(q) = 0)$ for any $k, q \in M$. $\alpha -$centralizer of a ring M is both left and right $\alpha -$ centralizer , where α is an additive mapping on M .

Definition (2.10):(3),(22)

A left (right) Jordan $\alpha -$ centralizer of a semi-ring M is an addittive mapping $R: M \to M$ which satisfy $R(k^2) + R(k)\, \alpha(k)' = 0, (R(k^2) + \alpha(k)'R(k) = 0)$ for any $k \in M, \alpha -$ Jordan centralizer of a ring M is both left and right Jordan $\alpha -$ centralizer, where α be additive mapping on M .

3. Main Results

To verify our main results, we must utilize the following.

Lemma (3.1):(4),(23)

If $V \not\subset Z(M)$ is a Lie-ideal of a $2 -$ tortion free prime semirig $M\ and\ k, q \in M$ such that $k\, V\, q = 0$, then $k = 0$ or $m = 0$.

From this we mean by V is a square closed $\text{lie} -$ ideal of M.

Lemma (3.2)

Let M be a $2-$tortion free prime semi-ring. Suppose that $F,G : VxV \rightarrow V$ biadditive mappings. If $F(k,q)\, w\, G(k,q) = 0$ for any $k,q,w \in V$, then $F(k,q)\, w\, G(u,v) = 0$ for any $k,q,u,v,w \in V$.

Proof:

$$F(k,q)\, w\, G(k,q) = 0 \qquad for\ all\ k,q,w \in V \qquad (*)$$

Replace k with $k + u$, we have

$$F(k + u,q)\, w\, G(k + u,q) = 0 \qquad for\ all\ k,q,w,u \in V$$

By using the additive of F and G

$$F(k,q)\, w\, G(u,q) = F(u,q)'\, w\, G(k,q)$$

Replace w by $2^4\, F(k,q)\, z\, G(u,q)$

$$(F(k,q)w\, 2^4 G(u,q))\, z\, F(k,q)\, w\, G(u,q) =$$
$$F(u,q)'\, w\, 2^4 G(u,q)\, z\, F(k,q)\, w\, G(k,q) = 0$$

by (*), we get

$$2^4 F(k,q)wG(u,q)zF(k,q)wG(u,q) = 0 \quad for\ all\ k,q,u,z \in V \qquad (**)$$

If $V \not\subset Z(M)$, by Lemma(3.1), we get

$$F(k,q)\, w\, G(u,q) = 0 \qquad for\ all\ k,q,u,w \in V$$

If $V \subset Z(M)$, multiply the relation (**) from the right by zt, where $t \in M$, we get

$$2^4 F(k,q)w\, G(u,q)\, z\, t\, F(k,q)wG(u,q)z = 0, \quad for\ all\ k,q,u,z,w \in V, t \in M$$

Since M is $2-$ tortion free prime semi-ring, we have

$$F(k,q)\, w\, G(u,q)\, z = 0 \qquad for\ all\ k,q,u,z,w \in V$$

If we multiply the relation by t an element of M, which is prime, and do a right multiplication, the result is

$$F(k,q)\, w\, G(u,q) = 0 \qquad for\ all\ k,q,u,w \in V$$

We can acquire the lemma's claim by exchanging $q\ for\ q + v$, in a way analogous to the one used above.

Theorem (3.3)

Let M be $2-$tortion free prime semi-ring. If R is left (right) Jordan $\alpha-$centralizer on V, then R is a left (right) $\alpha-$centralizer on V.

Proof:

$$R(k^2) + R(k)'\alpha(k) = 0 \qquad for\ all\ k \in V \qquad (1)$$

we replace k by $k + q$ when k,q in U, we get

$$R((k+q)^2) = R(k + q)\alpha(k + q)$$
$$R(k^2 + kq + qk + q^2) = R(k^2) + R(kq + qk) + R(q^2)$$
$$= R(k)\alpha(k) + R(kq + qk) + R(q)\alpha(q)$$
$$R(k+q)\alpha(k+q) = R(k)\alpha(k) + R(k)\alpha(q) + R(q)\alpha(k) + R(q)\alpha(q)$$

We get

$$R(kq+qk) + R(k)\alpha(q)' + R(q)\alpha(k)' = 0\ for\ all\ k,q \in V \qquad (2)$$

By replacing q with $2(kq + qk)$ and using (2), we get

$$2R(k(kq + qk) + (kq + qk)k) + 2R(k)\alpha(kq)' + 2R(k)\alpha(qk)' + R(kq + qk)\alpha(k)'$$
$$= 0$$
$$2R(k(kq + qk) + (kq + qk)k) = 2R(k)\alpha(kq) + 2R(k)\alpha(qk) + 2\, R(kq+qk)\alpha(k)$$

(3)

This can also be computed using an alternate way

$2R(k^2q + qk^2) + 4R(kqk) + 2R(k)\alpha(kq)' + 2R(q)\alpha(k^2)' = 0 \; for\ all\ k,q \in V$ (4)

From (3) and (4), we obtain

$R(kqk) + R(k)\alpha(qk)' = 0 \quad for\ all\ k,q \in V$ (5)

If we linearize (5), we get

$R(kqt + tqk) + R(k)\alpha(qt)' + R(t)\alpha(qk)' = 0 \quad for\ all\ k,q,t \in V$ (6)

Since V is a square closed Lie-ideal, we have

$$2^4(kqtqk + qktkq) \in V.$$

Now we shall compute $f = 2^4R(kqtqk + qktkq)$ in two different ways, using (5) we have

$f + 2^4R(k)\alpha(qtqk)' + R(q)\alpha(ktkq)' = 0 \quad for\ all\ k,q,t \in V$ (7)

Using (6) we have

$f + 2^4R(kq)\alpha(tqk)' + R(qk)\alpha(tkq)' = 0 \quad for\ all\ k,q,t \in V$ (8)

Comparing (7) and (8)

$R(k)\alpha(qtqk)' + R(q)\alpha(ktkq)' + R(kq)\alpha(tqk) + R(qk)\alpha(tqk) = 0$

$(R(kq) + R(k)\alpha(q)')\alpha(tqk) + (R(qk) + R(q)\alpha(k)')\,\alpha(tkq) = 0$

Introducing a additive mapping,

$$G(k,q) = R(kq) + R(k)\alpha(q)',$$

we arrive at

$$G(k,q)\alpha(tqk) + G(q,k)(tkq) = 0$$

By Lemma (2.5)

$G(k,q)\alpha(tqk) = G(q,k)'\alpha(tkq)$ (9)

We can be rewritten equality (2)in this notation as

$$G(k,q) + G(q,k)' = 0.$$

Using equality (9) and this fact, we obtain

$G(k,q)\alpha(\,t\,[k,q]) = 0 \quad for\ all\ k,q,t,z \in V$ (10)

Now using Lemma (3.2), we have

$G(k,q)\alpha(\,z\,[u,v]) = 0 \quad for\ all\ k,q,z,u,v \in V$ (11)

(i) If V is non commutative

Since α is surjective and using Lemma (3.1), we have

$$G(k,q) = 0 \quad for\ all\ k,q \in V$$

(ii) If V is commutative and $V \not\subset Z(M)$

Compute $N = 2^4 R(kqzqk)$ in two different ways. Using (5), we have

$N + 2^4 R(k)'\alpha(qzqk) = 0 \quad for\ all\ k,q,z \in V$ (12)

$N + 2^4 R(km)'\alpha(zmk) = 0 \quad for\ all\ k,q,z \in V$ (13)

From (12) and (13), we arrive at

$$R(kq)\alpha(zqk) + R(k)'\alpha(qzqk) = 0$$

$$(R(kq) + R(k)'\alpha(q))\alpha(zqk) = 0$$

$G(k,q)\alpha(zqk) = 0 \quad for\ all\ k,q,z \in V$ (14)

Let $\psi(k,q) = \alpha(qk)$, it's clear that ψ is additive mapping, therefore

$$G(k,q)\alpha(z)\psi(k,q) = 0 \qquad for\ all\ k,q,z \in V$$

Using Lemma (3.2), we have

$$G(k,q)\alpha(z)\psi(u,v) = 0 \qquad for\ all\ k,q,z,u,v \in V$$

Implies that

$$G(k,q)\alpha(zuv) = 0 \qquad for\ all\ k,q,z,u,v \in V \tag{15}$$

Replacing $\alpha(v)$ with $2G(k,q)\alpha(z)$, using Lemma (3.1) and M is prime semi-ring, we have

$$G(k,q)\alpha(z) = 0 \qquad for\ all\ k,q,z \in V$$

Using Lemma (3.1)

$$G(k,q) = 0 \qquad for\ all\ k,q \in V$$

(i) If $V \subset Z(M)$

Multiplying relation (15) on the right by t, where t $\in M$ and since M is a prime, we can obtain the result.

$$G(k,q) = 0 \qquad for\ all\ k,q \in V$$

If $R(k^2)+\alpha(k)'R(k)=0$, reaching the conclusion of the theorem with the same procedure as before completes the proof.

Lemma (3.4)

Let M be a $2-$tortion free prime semi$-$ring, $H,\alpha: M \to M$, H is $(\alpha,\alpha)-$derivation on $V\ and\ a \in V$ some fixed element, where α is automorphism of V, such that $\alpha(V)=V$ then

(ii) $H(k)H(q) = 0$ for any $k,q \in U$ implies $H = 0$ on V.

(iii)$a\alpha(k) + \alpha(k)'a \in Z(V)$ for any $k \in V$ implies $a \in Z(V)$.

Proof:

(i) $H(k)\alpha(q)H(k) = H(k)H(qk) + H(k)'H(q)\alpha(k)$

$$H(k)(H(q)\alpha(k) + \alpha(q)H(k)) + H(k)'H(q)\alpha(k) = 0$$

$$H(k)H(q)\alpha(k) + H(k)\alpha(q)H(k) + H(k)'H(q)\alpha(k) = 0$$

By hypothesis, and M is inverse semi-ring, we get

$$H(k)\alpha(q)H(k) = 0$$

Since α is automorphism of V, such that $\alpha(V)=V$, we get

$$H(k)\,V\,H(k) = 0 \qquad for\ all\ k \in V$$

If $V \not\subset Z(M)$, and α is automorphism of V, Lemma (3.2) we have $H = 0\ on\ V$.

If $V \subset Z(M)$

$$H(k)tH(k) = 0 \qquad for\ all\ k \in V, t \in M$$

So, by primness of M, we have

$$H = 0\ on\ V$$

(ii) Define $H(k) = a\alpha(k) + \alpha(k)a'$

It is easy to see that H is a $(\alpha,\alpha)-$derivations, since $H(k) \in Z(V)$ for any $k \in V$, we have $H(q)\alpha(k) = \alpha(k)H(q)$ and also $2H(qz)\alpha(k) = 2\,\alpha(k)H(qz)$

Since M is prime, we get

$$H(q)\alpha(zk) + \alpha(q)H(z)\alpha(k)$$

$$= \alpha(k)H(q)\alpha(z) + \alpha(kq)H(z)$$

$$H(q)(\alpha(z)\alpha(k) + \alpha(k)\alpha(z)') = H(z)(\alpha(q)\alpha(k)' + \alpha(k)(q))$$

$$H(q)[\alpha(z),\alpha(k)] = H(z)[\alpha(q),\alpha(k)]$$

Since α is automorphism, take $\alpha(z) = a$. Obviously $H(a) = 0$, so, we obtain by (i)

$$H(q)H(k) = 0$$

By virtue of (i) we get $H = 0$ and hence $a \in Z(M)$.

Lemma (3.5)

Let M be a $2-$tortion free prime semi $-$ ring, R and α are additive mappings on M, and $a \in V$ some fixed element. If $R(k) = a\,\alpha(k) + \alpha(k)a$ and $R(k\ o\ q) + R(k)o\ \alpha(q)' = 0$ and $R(k\ o\ q) + \alpha(k)' o R(q) = 0$ for any $k, q \in V$ then "$a \in Z(V)$, where α is a surjective endomorphism of V.

Proof:

By hypothesis

$$R(kq + qk) = R(k)\alpha(q) + \alpha(q)R(k) \qquad for\ all\ k, q \in V$$

$$R(kq) + R(qk) = R(k)\alpha(q) + \alpha(q)R(k) \qquad for\ all\ k, q \in V$$

$$R(kq) + R(qk) = a\alpha(kq) + \alpha(kq)a + a\alpha(qk) + \alpha(qk)a$$

$$= a\alpha(k)\alpha(q) + \alpha(k)\alpha(q)a + a\alpha(q)\alpha(k) + \alpha(q)\alpha(k)a$$

$$R(k)\alpha(q) + \alpha(q)R(k) = a\alpha(k)\alpha(q) + \alpha(k)a\alpha(q) + \alpha(q)a\alpha(k) + \alpha(q)\alpha(k)a$$

$$+ a\alpha(k)\alpha(q) + \alpha(k)\alpha(q)a + a\alpha(q)\alpha(k) + \alpha(q)\alpha(k)a$$

$$= a\alpha(k)\alpha(q) + \alpha(k)a\alpha(q) + \alpha(q)a\alpha(k) + \alpha(q)\alpha(k)a$$

$$(a + a')\alpha(k)\alpha(q) + \alpha(q)\alpha(k)(a + a') + \alpha(k)\alpha(q)a + a\alpha(q)\alpha(k) + \alpha(k)a'\alpha(q) + \alpha(q)a'\alpha(k) = 0$$

Since $a + a' \in Z(V)$

$$\alpha(k)\alpha(q)(a + a' + a) + (a + a' + a)\alpha(q)\alpha(k) + a\alpha(q)\alpha(k) + \alpha(k)a'\alpha(q) = 0$$

$$\alpha(k)\alpha(q)a + \alpha(k)a'\alpha(q) + a\alpha(q)\alpha(k) + \alpha(q)a'\alpha(k) = 0$$

$$\alpha(k)(\alpha(q)a + a'\alpha(q)) + (\alpha(q)a + a'\alpha(q))\alpha(k)' = 0$$

But α is a surjective

$$a\alpha(k) + \alpha(k)a' \in Z(V)$$

By Lemma (3.4) (ii), we get $a \in Z(V)$

$$R(k\ o\ q) + R(k)o\ \alpha(q)' = 0 \text{ and } R(k\ o\ q) + \alpha(k)'o\ R(q) = 0.$$

Lemma (3.6)

Let M be a $2-$tortion free prime semi-ring, and R, α are additive mappings on M, R satisfies $R(k \text{ o } q) + R(k)\text{o}\ \alpha(q)' = 0$ and $R(k\ o\ q) + \alpha(k)'\ o\ R(q) = 0$ for any $k, q \in V$, then $R(z) \in Z(V)$ for any $z \in Z(V)$, where α is a surjective endomorphism of V.

Proof:

$$R(kq + qk) + R(k)\alpha(q)' + \alpha(q)'R(k) = 0$$

$$R(kq + qk) + \alpha(k)'R(q) + R(q)\alpha(k)' = 0.$$

because $R(z) \in Z(V)$

Take any $t \in Z(U)$ and denote $a = R(t)$

$$2R(tk) = R(tk + kt) = R(t)\alpha(k) + \alpha(k)R(t)$$

$$= a\alpha(k) + \alpha(k)a$$

A simple check reveals that $M(k) = 2R(tk)$ is satisfies

$$M(k\ o\ q) = 2R(t(kq + qk) = 2R(tkq + qtk)$$

$$= 2R(tk)\alpha(q) + 2\,\alpha(q)R(tk)$$

$$= M(k)\alpha(q) + \alpha(q)M(k)$$
$$= M(k)o\alpha(q)$$
$$M(k\ oq) = 2R(t(kq + qk) = 2R(k(tq) + (tq)k)$$
$$= 2\alpha(k)R((tq) + 2R(tq)\alpha(k)$$
$$= \alpha(k)M(q) + M(q)\alpha(k)$$
$$= \alpha(k)\ o\ M(q)$$
$$M(k\ o\ q) = M(k)\ o\ \alpha(q) = \alpha(k)\ o\ M(q)\ \ fon\ all\ k,q \in M$$

By Lemma (3.5), we have $R(t) \in Z(M)$.

Theorem (3.7)

Let M be $2-$tortion free prime semi $-$ ring and $R,\alpha: M \to M$ additive mappings, R satisfies $R(k\ o\ q) + R(k)o\alpha(q)' = 0$ *and* $R(k\ o\ q) + \alpha(k)'oR(q) = 0$ *for all* $k,q \in V$ then R is $a\ \alpha -$ centralizer on V, where α is an automorphism of V, $R(u) \in V$, for any $u \in V$, and $\alpha\big(Z(V)\big) = Z(V)$.

Proof :

Since U is a square closed Lie $-$ ideal of M, and by Lemma (2.5), we get

$$2R(kq + qk) = 2R(k)\alpha(q) + 2\alpha(q)R(k)$$
$$= 2\alpha(k)R(q) + 2R(q)\alpha(k)$$

If V is a commutative, we have

$$R(r^2) = R(r)\alpha(r) = \alpha(r)R(r)$$

If V is a non-commutative

Replace q by $2kq + 2qk$ in (2), we get,

$$4R(k)\alpha(kq + qk) + 4\alpha(kq + qk)R(k)$$
$$= 4\alpha(k)R(kq + qk) + 4R(kq + qk)\alpha(k)$$
$$4R(k)\alpha(k)\alpha(q) + 4R(k)\alpha(q)\alpha(k) + 4\alpha(k)\alpha(q)R(k) + 4\alpha(q)\alpha(k)R(k) =$$
$$4\alpha(k)R(k)\alpha(q) + 4\alpha(k)\alpha(q)R(k) + 4R(k)\alpha(q)\alpha(k) + 4\alpha(q)R(k)\alpha(k)$$

By using the property of $2-$tortion free semi $-$ ring, we obtain

$$R(k)\alpha(k)\alpha(q) + \alpha(q)\alpha(k)R(k) + \alpha(k)'R(k)\alpha(q) + \alpha(q)R(k)\alpha(k)' = 0$$

Now it follows that

$$[R(k),\alpha(k)]\alpha(q) = \alpha(q)[R(k),\alpha(k)] \qquad for\ all\ k,q \in V$$

but α is surjective, then we get

$$[R(k),\alpha(k)] \in Z(V) \qquad for\ all\ k,q \in V$$

The next goal is to show that $[R(k),\alpha(k)] = 0\ \ for\ all\ k \in V.$

Take any $t \in Z(U)$

$$4R(tk) = 2R(tk + kt) = 2R(t)\alpha(k) + 2\alpha(k)R(t)$$
$$= 2R(k)\alpha(t) + 2\alpha(t)R(k)$$

Using Lemma (3.6), we get

$$R(tk) = R(k)\alpha(t) = R(t)\alpha(k) \qquad for\ all\ k,t \in V$$
$$4[R(k),\alpha(k)]\alpha(t) = 4R(k)\alpha(kt) + 4\alpha(k)'R(k)\alpha(t)$$
$$= 4R(k)\alpha(tk) + 4R(k)\alpha(t)\alpha(k)' = 0$$

Since $\alpha(Z(V)) = Z(V)$, and $[R(k),\alpha(k)]$ itself is central element, By Lemma (3.1), we get our goal.

$$2R(k^2) = R(kk + kk) = R(k)\alpha(k) + \alpha(k)R(k)$$
$$= 2R(k)\alpha(k) = 2\alpha(k)R(k).$$

By Theorem 3.3, we get our result.

4. Conclusion

In this work, we extend certain results of α-centralizers and Jordan α-centralizers on lie ideals of prime rings to prime inverse semirings. We got the output R is a left (right) $\alpha -$ centralizer on V. If it where α is an automorphism of V,$R(u) \in V$, for any $u \in V$, and $\alpha\big(Z(V)\big) = Z(V)$. We also get the following output R is a $\alpha -$ centralizer on V

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Conflict of Interest

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