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Some Results on Double Centralizer for Prime and Semiprime Γ- rings

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Abstract

The goal of this work, is to examine the concept of a double centralizer (T, S), and double Jordan centralizer on prime and semiprime Γ-rings, this is done by studying examples ,remarks and results related to that concepts and looking for the conditions under which T equal S, we prove the results, the first result , let A be a semiprime Γ-ring and T is a left centralizer, S is a right centralizer, and they fulfilling $x\ \alpha$ T(y) = S (x) α y, for each x ∈ A, $\alpha \in \Gamma$, thence (T,S) is a double centralizer. The second, let A be a prime Γ-ring, U be a not equal zero ideal of A, such that, T is a left centralizer, S is a right centralizer, and fulfilling x αT(y) = S (x) α y, for each x, y ∈ U, $\alpha \in \Gamma$, thence (T, S) is a double centralizer. The third, let A be a prime Γ-ring, U be a not equal zero ideal of A and we get, if T=S on U, thence T=S on A.

Keywords: Prime Γ-rings, Semiprime Γ-rings, centralizer, Jordan centralizer, double centralizer, double Jordan centralizer.

1. Introduction

Barnes (1) defined Γ-ring. Let A and Γ be two additive abelian groups. It there is a mapping $(x, \alpha, y) \rightarrow (x\ \alpha\ y)$ of $A\ \times\ \Gamma \times A \rightarrow A$, satisfying the following, for any $x, y, z \in$ A and $\alpha, \beta \in \Gamma$.

i. $(x+y)\alpha z = x\alpha z + y\alpha z$,
$x(\alpha+\beta)y = x\alpha y + x\beta y$,
$x\alpha(y+z) = x\alpha y + x\alpha z$,

ii. $(x\alpha y)\beta z = x\alpha(y\beta z)$, thence A is named a Γ-ring.

OZDEN et al. (2) defined the subring. A subring of Γ-ring A is additive subgroup S of A such that $S\Gamma S \subseteq S$. Let A be a Γ-ring, thence A is named a commutative gamma-ring if, $x\alpha y = y\alpha x$, holds for any $x, y \in A$ and $\alpha \in \Gamma$, Kandamar et al. (3). A subset U of the Γ-ring A is a right (left) ideal of A if U is an additive subgroup of A and $U\Gamma A = \{a\alpha x\text{: } a \in U, \alpha \in \Gamma, x \in A\}$ ($A\Gamma U$) is contained in U. If U is both a left and a right ideal, thence U is a two-sided ideal, or simply is an ideal of A. Barnes (1). A Γ-ring A is named prime if $m\Gamma A\Gamma n = (0)$ with $m, n \in A$ implies $m = 0$ or $n = 0$ and semiprime if $m\Gamma A\Gamma m = 0$ with $m \in A$ implies $m = 0$ (4, 5). An ideal P of a gamma-ring A is prime ideal if for any ideals N,M⊆A, NΓM⊆ P implies N⊆ P or M⊆ P, Kyuno (5). A gamma-ring A is said to be prime Γ-ring if the zero ideal is prime ideal, Kyuno (5). Let A be a Γ-ring, thence A is named n-torsion free if $n\,x = 0$, yields $x = 0$, for every $x \in$ A, where n is positive integer, Chakraborty et al. (6). Let A be a gamma-semiring, an element $1 \in A$, is named unity for any $x \in A$ there are $\alpha \in \Gamma$ such that $x\,\alpha\,1 = 1\,\alpha\,x = x$, RAO (7). Özkum et al. (8) defined the derivation and (Jordan derivation), let A be a gamma-ring and $D : A \to A$ and additive map. Thence D is derivation (resp. Jordan derivation), if $D\,(m\,\alpha n\,) = D\,(m)\,\alpha\,n + m\alpha\,D\,(n)$ (resp. $D\,(m\,\alpha\,m) = D\,(m)\alpha\,m + m\alpha\,D(m)$), for any $m, n \in$ A and $\alpha \in \Gamma$, Özkum et al. (8). Every derivation of A, is Jordan derivation but the converse in general is not true, see Saleh (9). Barnes (1) defined the Γ-homomorphism. Let A and Y both be Γ-rings, and $\emptyset$ a map of A in to A. Thence $\emptyset$ is a Γ-homomorphism, if and only if $\emptyset(x\alpha y) = \emptyset(x)\alpha\,\emptyset\,(y)$, $for\ all x, y \in A$ and $\alpha \in \Gamma$. If $\emptyset$ is also one-to one and onto thence $\emptyset$ is a Γ-isomorphism. An additive mapping $\emptyset$ of Γ-ring A into a Γ-ring A' is named Jordan homomorphism if $\emptyset(x\alpha y + y\alpha x) = \emptyset(x)\alpha\emptyset(y) + \emptyset(y)\alpha\emptyset(x)$, for each $x, y \in$ A and $\alpha \in \Gamma$, Shaheen (10). Let A be a Γ-ring, a mapping d of A, to itself is named Γ-centralizing on a subset S of A if $[\,x, d\,(x)]_\alpha \in Z(A)$, for every $x \in S$ and $\alpha \in \Gamma$,in the special case when $[\,x, d\,(x)]_\alpha = 0$, hold for any $x \in$ S and $\alpha \in \Gamma$, the mapping d is named Γ-commuting on S , Sameer et al. (11). Many researchers have studied centralizers and derivations in prime and semiprime Γ- rings (12-21) and (22-30). The objective of this paper is to debate, double centralizer (T, S), and double Jordan centralizer on prime and semiprime Γ- rings, with fulfilling certain identities.

2. Preliminaries and Fundamentals

2.1 Definition

Ali et al. (18) Let A be a gamma-ring, for any $x, y \in$ A and $\alpha \in \Gamma$, the symbol $[r, t]_\alpha = r\,\alpha\,t - t\,\alpha\,r$, to symbolize the commutator. $T(r \circ t) = r\,\alpha\,t + t\,\alpha\,r$.

2.2 Lemma

Ali et al. (18) If A is a gamma-ring, for any $r, t, s \in$ A and $\alpha, \beta \in \Gamma$ thence:

I. $[r, t]_\alpha + [t, r]_\alpha = 0$

II. $[r + t, s]_\alpha = [r, s]_\alpha + [t, s]_\alpha$

III. $[r, t + s]_\alpha = [r, t]_\alpha + [r, s]_\alpha$

IV. $[r, t]_{\alpha+\beta} = [r, t]_\alpha + [r, t]_\beta$

V. $[r\beta t\,, s]_\alpha = r\beta[t, s]_\alpha + [r, s]_\alpha\,\beta t + r\,\beta\,s\,\alpha\,t - r\,\alpha\,s\,\beta\,t$.

2.3 Definition

Hoque 20(19) An additive mapping T: A → A is a left (right)centralizer, if $T(r\alpha\,t\,) = T\,(r)\,\alpha\,t$ ($T\,(r\,\alpha\,t\,) = r\,\alpha\,T\,(t)$) holds for any $r\,, t \in$ A and $\alpha \in \Gamma$. A centralizer is both a left and right centralizer.

2.4 Example

Let F be a field, and $D_2\ (F)$be a diagonal matrices 2 by 2 over F and $\Gamma = \{ \begin{bmatrix} 0 & 0 \\ 0 & n \end{bmatrix},\ n \in Z\}$, define $T: D_2\ (F) \to D_2\ (F)$ as

$T\left(\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}\right) = \begin{bmatrix} 0 & 0 \\ 0 & b \end{bmatrix}$, *for any* $a, b \in F$.

Thence T is a centralizer.

2.5 Definition

Hoque (19)An additive mapping $T: A \to A$, is Jordan left (right) centralizer, if $T(x\alpha x) = T(x)\alpha x$ $(T(x\alpha x) = x\alpha T(x)$,for any $x \in A$ and $\alpha \in \Gamma$.

2.6 Definition

Let A be gamma-ring, let $T, S: A \to A$, be an additive mappings, thence a mate (T, S) is named a double centralizer, if T is a left centralizer, S is a right centralizer, and they satisfy a balanced requirement, $x\alpha\, T(y) = S(x)\, \alpha y$, for any $x, y \in A$, $\alpha \in \Gamma$.

2.7 Definition

Let A be gamma-ring, and let $T, S: A \to A$, be an additive mapping, thence a mate (T, S) is named a double Jordan centralizer, if T is a left Jordan centralizer, S is a right Jordan centralizer, and they satisfy a balanced requirement, $(x\alpha\, T(x) = S(x)\, \alpha\, x)$, for any $x \in A$, $\alpha \in \Gamma$.

3. Main Results

In the following, we give the definition of commuting double centralizer:

3.1 Definition

Let A be a Γ-ring, and (T, S), be a double centralizer. Thence (T, S), is named commuting double centralizer, if T and S are commuting.

Now, we shall give an example for a commuting double centralizer.

3.2 Example

Let F be a field, and A be a Γ-ring of all triangular matrices of the from

$x = \{\begin{bmatrix} d & a & c & b \\ 0 & d & 0 & c \\ 0 & 0 & d & -a \\ 0 & 0 & 0 & d \end{bmatrix}, forall\ a, b, c, d\ \in F\}$, and $\Gamma = \{\begin{bmatrix} 0 & 0 & 0 & n \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, forall\ n \in Z\}$.

In connection to the frequent process of addition and multiplication and let T, S: A→A be additive mappings defined by

$T(x) = y\alpha x$ and $S(x) = x\alpha y$, for each $x, y \in A$ and $\alpha \in \Gamma$.

Where;

$y = \begin{bmatrix} 0 & 0 & 0 & b \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, for\ all\ b \in\ F$.

It is clear that T and S are commuting double centralizer.

In the following results, we give some certain conditions to obtain (T,S) is a double centralizer, where T and S are from A to A.

3.3 Theorem

Let A be a semiprime Γ-ring and T, S: A→ A be a mapping fulfilling.

$x\alpha T(y) = S(x)\alpha y$, for each $x, y \in A$ and $\alpha \in \Gamma$. (1)

Thence (T, S) is a double centralizer.

Proof: We need to show that T, S are additive mapping, and
$T\ (x\ \alpha\ y) =\ T\ (x)\ \alpha\ y, for\ all\ x, y\ \in A\ and\ \alpha \in \Gamma.$
$S\ (x\ \alpha\ y) =\ x\ \alpha\ S\ (y), for\ all\ x, y \in A, \alpha \in \Gamma.$
Now replace y by $y + z$ in (1), we imply
$x\alpha\ T(y+z) = S(x)\alpha y + S\ (x)\alpha z,\ for\ each\ x, y, z \in A\ and\ \alpha \in \Gamma.$
Hence
$x\alpha(T(y+z) -\ T\ (y) - T(z)) = 0, for\ all\ x, y, z \in A\ and\ \alpha \in \Gamma.$
By the semiprimeness of A, we imply
$T(y+z)\ =\ T(y) +\ T\ (z), for\ each\ \ y, z \in\ A.$
Similarly, we can show that
$S(x+y) = S\ (x) + S\ (y), for\ each\ \ x, y, \in\ A.$
Now, replacing y with $y\beta z$ in (1) we obtain
$x\ \alpha\ (T\ (y\ \beta z) - T\ (y)\ \beta z)\ = 0, for\ each\ \ x, y, z \in A\ and\ \alpha,\ \beta \in \Gamma.$
By the semiprimeness of A, we imply
$T\ (y\ \beta\ z)\ =\ T\ (y)\ \beta\ z, for\ each\ \ y, z\ \in\ A\ and\ \beta \in \Gamma.$
Similarly, we can show
$S\ (x\ \alpha\ y)\ =\ x\ \alpha\ S\ (y)$, for each $x, y \in A\ and\ \alpha \in \Gamma.$
Thence (T, S) is a double centralizer.
Now, we give some results which make T=S under different conditions, where (T,S) is double centralizer.

3.4 Theorem

Let A be a prime Γ-ring, U be a not equal zero ideal of A. Let $T, S: A \rightarrow A$ be additive mappings such that T is a left centralizer, S is a right centralizer and they gratify $x\alpha T(y) = S(x)\alpha y$, for each $x, y \in U\ and\ \alpha \in \Gamma$. Thence (T,S) is a double centralizer.

Proof: We have
$x\ \alpha\ T(y)\ =\ S\ (x)\ \alpha\ y, for\ each\ \ x, y\ \in\ U, \alpha\ \in\ \Gamma.$ (2)
Replace x with $x\beta r$ in (2) when $x \in U, \beta \in \Gamma\ and\ r \in A$, we imply
$x\ \beta\ (r\ \alpha\ T(y) - S\ (r)\ \alpha\ y) = 0, for\ each\ r \in A, x, y \in\ U and\ \alpha\ , \beta \in \Gamma.$
i.e.
$x\ \gamma\ A\ \beta\ (r\ \alpha\ T(y) -\ S\ (r)\alpha\ y) = 0\ , for\ each\ \ r \in A, x, y \in U and\ \alpha\ , \beta, \gamma\ \in\ \Gamma.$
By primeness of A and since U be a not equal zero ideal of A, we imply
$r\alpha T(y) = S(r)\alpha y, for\ each\ r \in A,\ \ y \in U\ and\ \alpha\ \in \Gamma.$ (3)
Replacing y with $t\sigma y$ in (3), where $t\ \in\ A,\ y\ \in\ U$, and $\sigma \in \Gamma$.
$(r\ \alpha\ T(t) -\ S\ (r)\alpha\ t\ )\sigma\ y = 0\ , for\ each\ \mathrm{t}, r \in A,\ \ y \in U\ and\ \alpha, \sigma \in\ \Gamma.$
Implies that
$(r\alpha T(t) - \mathrm{S(r)\alpha t})\sigma \mathrm{U}\delta A = 0, for\ each\ t, r \in \mathrm{A}$ and $\alpha, \sigma, \delta \in \Gamma.$
By the primeness of A, we imply
$r\ \alpha\ T(t) =\ S\ (r)\alpha\ t,$ for each $t, r\ \in\ A, and, \alpha\ \in\ \Gamma.$

3.5 Theorem

Let A be a prime gamma-ring, U be a not equal zero ideal of A, and (T,S) be a double centralizer. If T=S on U, thence T=S on A.

Proof: We have
$T\ (x) =\ S\ (x), for\ each\ \ x\ \in\ U.$ (4)
By replacing x with $r\alpha x$ in (4), when $r \in \mathrm{A}$, $x \in \mathrm{U}$ and $\alpha \in \Gamma$, we imply

$T(r)\alpha x = r \alpha S(x) = r\alpha T(x), for\ each \in A,\ x \in U\ and\ \alpha \in \Gamma$. (5)

Since (T, S) are a double centralizer, (5) leads to

$T(r)\alpha x = S(r)\alpha x, for\ each\ x \in U, r \in A, and\ \alpha \in \Gamma$.i.e.

$(T(r) - S(r))\alpha U\beta A = 0,$ *for each* $r \in$ A, and $\alpha, \beta \in \Gamma$.

Since A is a prime Γ-ring and U be a not equal zero ideal of A, we imply T = S.

From Theorem above, we imply the following:

3.6 Corollary

Let A be a prime gamma-ring, U be an ideal of A and (T, S) be a double centralizer. If, T = S = 0 on U, thence T = S = 0 on A.

In the following theorem, we shall prove that T=S in case T acts as a homomorphism on A.

3.7 Theorem

Let A be a semiprime gamma-ring and let (T,S) be a double centralizer, if T acts as a homomorphism on A, thence T = S.

Proof: We have

$T(x\alpha y) = T(x)\alpha y, for\ each\ x, y \in A\ and\ \alpha \in \Gamma$.

Since T is acts homomorphism on A, thence

$T(x)\, \alpha\, T(y) = T(x)\, \alpha\, y, for\ any\ x, y \in A\ and\ \alpha \in \Gamma$. (6)

On the other hand;

$x\, \alpha\, T(y) = S(x)\, \alpha\, y, for\ all\ x, y \in A\ and\ \alpha \in \Gamma$. (7)

The substation $T(x)$ for x in (7), gives

$T(x)\, \alpha\, T(y) = S(T(x))\, \alpha\, y, for\ each\ x, y \in A\ and\ \alpha \in \Gamma$. (8)

By comparing (6) with (8), we arrive at $(S(T(x)) - T(x))\, \alpha\, y = 0$

Multiply from the right by $(S(T(x)) - T(x))$, we get

$(S(T(x)) - T(x))\beta\, y\, \alpha\, (S(T(x)) - (T(x)) = 0, for\ each\ x, y \in A\ and\ \alpha, \beta \in \Gamma$.

By semiprimeness of A, we have

$S(T(x)) = T(x), for\ each\ x \in A$. (9)

From (9) and using (7), we imply

$T(x)\alpha T(y) = T(x)\alpha S(y), for\ each\ x, y \in A\ and\ \alpha \in \Gamma$. (10)

Replace x by $x\beta z$ and y by $y\sigma w$ in (7) and using (10), we arrive at

$x\beta z\, \alpha T(y)\sigma S(w) = S(x\, \beta\, z)\, \alpha\, y\, \sigma\, w, for\ each\ x, y, z, w \in A\ and\ \alpha, \beta, \sigma, \in \Gamma$. (11)

Thence by (11), we imply

$x\beta S(z)\alpha\, y\, \sigma\, S(w) = S(x\, \beta\, z)\alpha y\sigma w, for\ all\ x, y, z, w \in A\ and\ \alpha, \beta, \sigma \in \Gamma$.

Whence it follows that

$x\beta S(z)\alpha y\sigma(S(w) - w) = 0, for\ each\ x, y, z, w \in A and\ \alpha, \beta, \sigma \in \Gamma$. (12)

The substitution on S (w)− w for x in (12), gives us

$(S(w) - w)\beta S(z)\alpha y\sigma(S(w) - w) = 0,$ for each $y, z, w \in A\ and\ \alpha, \beta, \sigma \in \Gamma$.

Right multiplication the above relation by S(z), yields

$(S(w) - w)\beta S(z)\alpha y\sigma(S(w) - w)\gamma S(z) = 0, for\ all\ y, z, w \in A\ and\ \alpha, \beta, \sigma, \gamma \in \Gamma$.

By the semiprimeness of A, we obtain

$(S(w) - w)\beta S(z) = 0,$ *for each* $w, z \in$ A and $\beta \in \Gamma$.

This gives

$S(w)\, \beta\, S(z) = w\, \beta\, S(z), for\ each\ w, z \in A\ and\ \beta \in \Gamma$. (13)

From (7) and (13), we obtain

$x\,\alpha\,T(y) = S(x)\,\alpha\,S(y) = x\,\alpha\,S(y), for\ each\ x, y \in A\ and\ \alpha \in \Gamma.$
Of course, we have also,
$x\alpha(T(y) - \mathrm{S(y)}) = 0,$ *for each* $x, y \in$ A, and $\alpha \in \Gamma$.
By the semiprimeness of A, we imply T=S.
In the following theorem, we gives a relation between T and S, where (T,S) is a double centralizer on prime Γ-ring.

3.8 Theorem

Let A be a prime Γ-ring, and U be a not equal zero ideal of A, we imply (T,S) be a double centralizer. If $T(r\,\alpha\,x) = S(r)\alpha x\ for\ all\ r \in A, x \in U, then\ T = S.$

Proof: We have
$T(r\alpha x) = T(r)\alpha x = S(r)\alpha x$ *for each* $r, t \in$ A , $x \in$ U and $\alpha \in \Gamma$.
This reduces to
$\big(T(r) - S(r)\big)\alpha x = 0,$ *for each* $r \in$ A , $x \in$ U and $\alpha \in \Gamma$. (14)
Replacing x with $t\beta x$ in (14), when $t \in$ A, $x \in$ U, $\alpha \in \Gamma$, leads to
$(T(r) - S(r))\alpha t\beta x = 0,$ *for each* $r \in A, x \in U\ and\ \alpha, \beta \in \Gamma$.
i.e.
$(T(r) - S(r))\alpha A\,\beta U =,$ for each $r \in$ A and $\alpha , \beta \in \Gamma$.
Since A is a prime gamma-ring, and U be a not equal zero ideal, we have T = S.

3.9 Theorem

Let A be a prime Γ-ring, and (T, S) be a double centralizer, if T acts as a not equal zero Jordan homomorphism $\big(T(x\alpha x) = T(x)\alpha\,T(x)\big)$ for each $x \in A\ and\ \alpha \in \Gamma$. Thence T=S=id.

Proof : We have
$T(x\alpha x) = T(x)\alpha x,$ for each $x \in$ A and $\alpha \in \Gamma$.
Thence from above relation and since T is acts as Jordan homomorphism.
Yields,
$T(x)\,\alpha\,(T(x) - x) = 0,$ for each $x \in A\ and\ \alpha \in \Gamma$. (15)
Replace x by $x\beta\,y$ in (15), we imply
$T(x)\beta\,y\,\alpha\,(T(x) - x)\beta\,y = 0,$ for each $x, y \in A\ and\ \alpha, \beta \in \Gamma$. (16)
Linearization (16), we imply
$(T(x)\,\beta\,y\,\alpha\,(\,T(x) - x)\beta\,z + T(x)\,\beta\,z\,\alpha(\,T(x) - x)\beta\,y = 0$ (17)
Now, replacing z by $y\sigma z$ in (17) and using (16), we obtain
$(T(x)\,\beta\,y\,\alpha\,A\,\gamma\,(\,T(x) - x)\beta y,$ *for each* $x, y \in A, \alpha, \beta, \gamma \in \Gamma$.
By the primeness of A, we imply
$T(x) = x,$ for each $x \in$ A. (18)
Otherwise, T=0. From $x\alpha T(y) = \mathrm{S(x)}\alpha y$, and by (18), we imply T = S = id.

4. Conclusion

This work is to discuss double centralizer (T, S), and double Jordan centralizer on prime and semiprime Γ- rings, with fulfilling certain identities. We prove that; when T is a left centralizer, S is a right centralizer, and they fulfilling $x\alpha T(y) = S(x)\alpha y$, for each $x, y \in U\ and\ \alpha \in \Gamma$. Then (T,S) is a double centralizer. Also if (T, S) be a double centralizer, T acts as a homomorphism on A, then T=S, and if T acts as a not equal zero Jordan homomorphism $\big(T(x\alpha x) = T(x)\alpha\,T(x)\big)$ for each $x \in A\ and\ \alpha \in \Gamma$. Then T = S = id.

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Conflict of Interest

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